Children's reasoning and mathematical achievement

Terezinha Nunes
Department of Educational Studies
University of Oxford

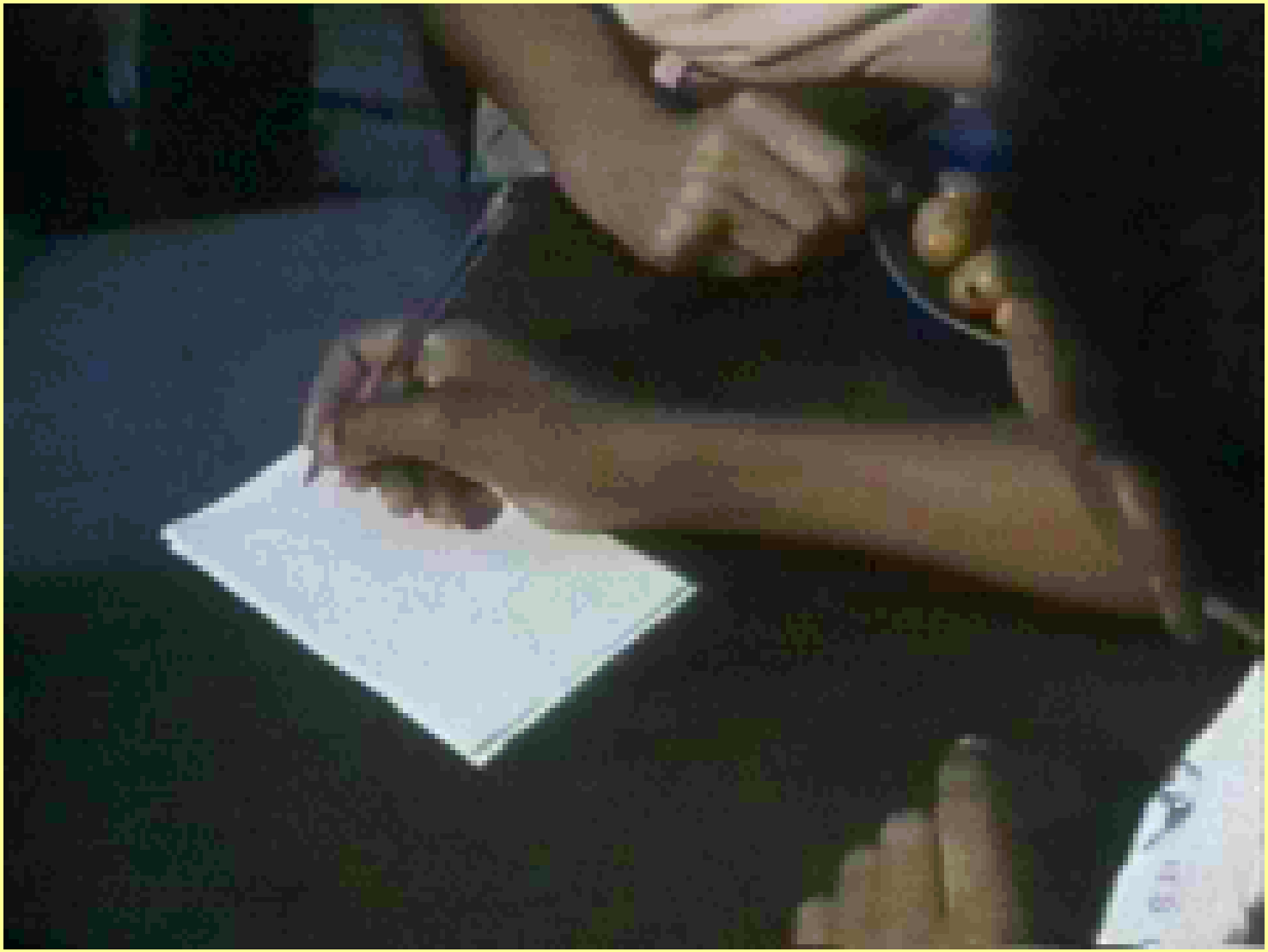
The team

- Peter Bryant
- Deborah Evans
- Daniel Bell
- Selina Gardner
- Julia Carraher
- Adelina Gardner

- The role of logic
  - What happens if children are taught how to solve a certain type of problem but they can’t see the logic of it?
- What logical moves do children have to connect with the procedures that they are taught in primary school?
- The relation between children’s logic and mathematics learning
  - a longitudinal study
  - an intervention study

Three examples from procedures disconnected from the learner's logic

- First example is from Brazil: the disconnection between oral and written arithmetic
- Two examples from English primary school
  - Subtraction by complementary addition
  - Using the number line to solve problems



En la vida diez, en la escuela zero

The lack of links between logic and procedures

- Brazilian children who participate in the informal economy use oral arithmetic
- In school, they are taught written arithmetic
- They receive no teaching to help them make a connection between the two

Subtraction by complementary addition

- The child was taught what she should do: she knew that she could count up instead of counting down to find the result of a subtraction
- Once she counted up, she did not know what the number she obtained had to do with the problem
- She was not led by the teacher’s pressure (even though you counted up 3?)

Using the number line to solve problems

Lack of connection between her reasoning and the number line

- The difficulties of the number line
- The need to work on connections between the children's reasoning and different ways of representing mathematics

What is the connection between logic and mathematics learning?

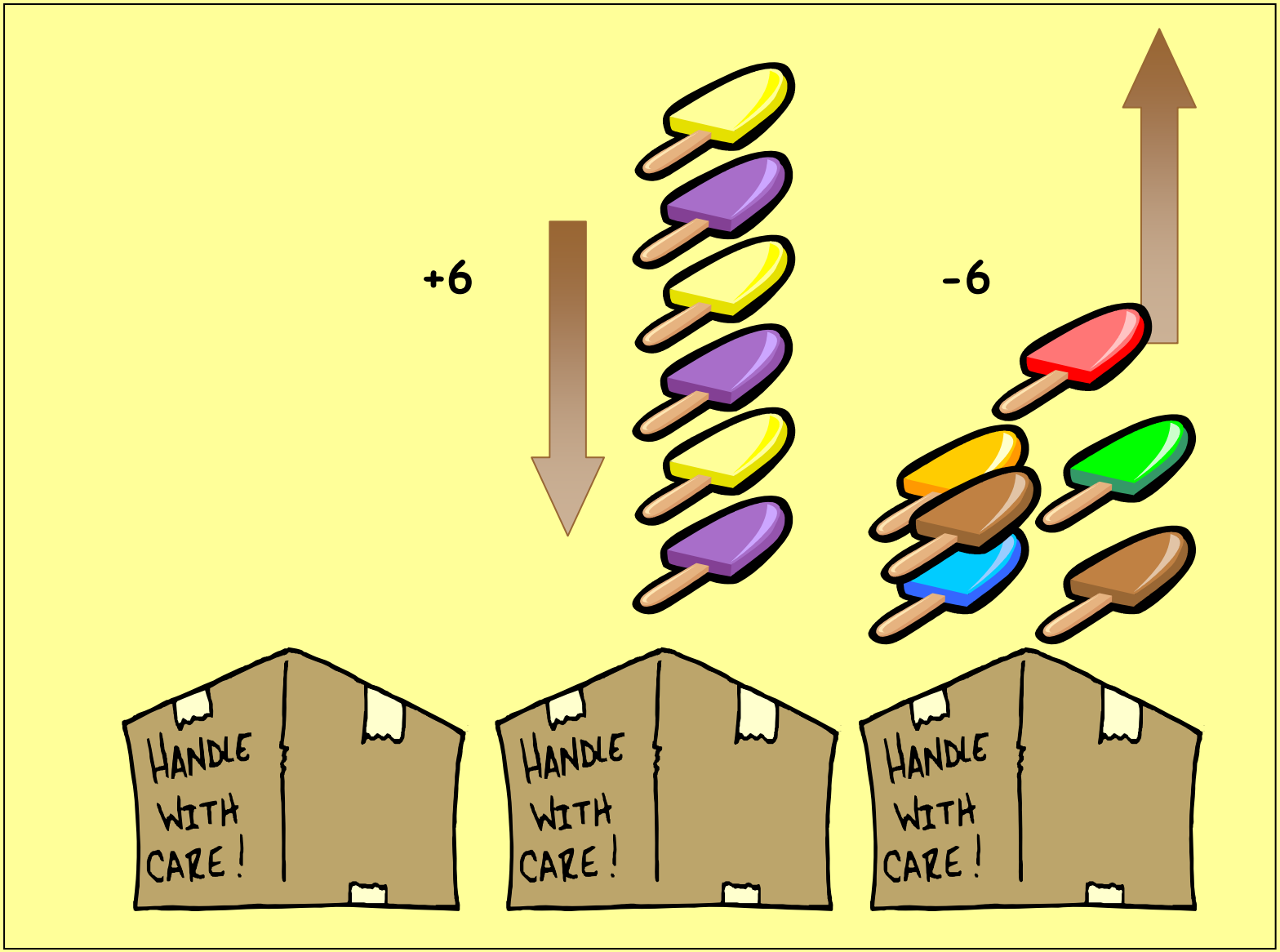
- Our hypothesis is that logic forms the basis for learning
- It is vital that children make connections between logic and the mathematics that they learn in school
- This means that their logical understanding at the start of school is a good indicator of how well they will learn mathematics

What is the connection between logic and mathematics learning?

- Logical understanding is NOT the same as general intelligence in the traditional sense measured by IQ tests
- In a longitudinal study, it is possible to show that children's logical understanding relates to their mathematics learning even after controlling statistically for their results in an intelligence test

The logico-mathematical schemas we investigated

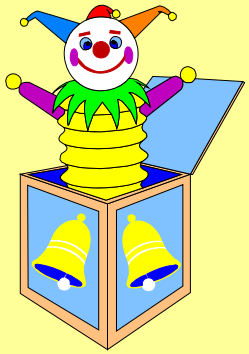
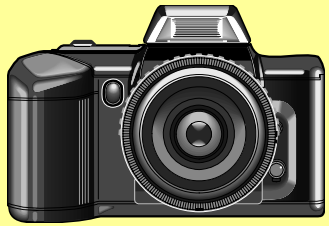
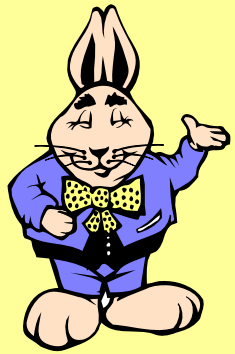
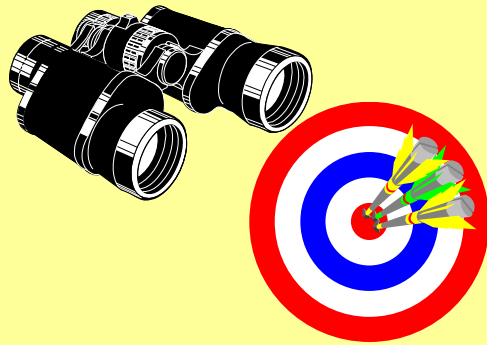
- The inverse relation between addition and subtraction
  - A number doesn’t change if we add and subtract the same amount from it
  - Solving inverse addition and subtraction problems



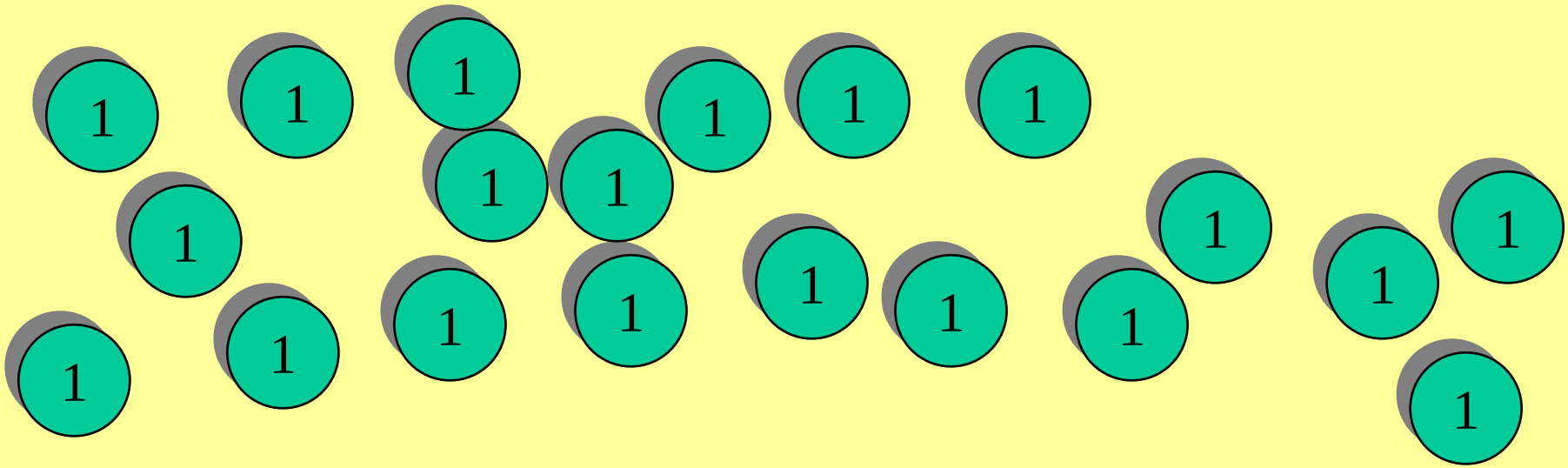
Only about 50% of the answers given by children at the end of Year 1 (mean age 6y3m) are correct

- Additive composition of numbers
  - Any number can be formed by the addition of two other numbers
- We developed a Shop Task to assess children's understanding of additive composition

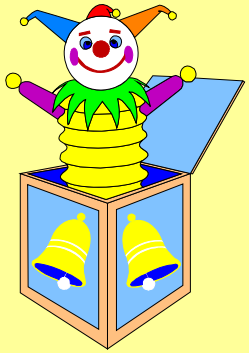
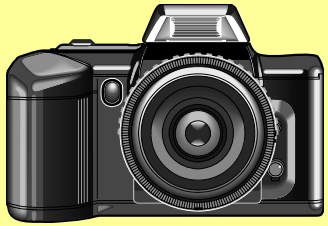
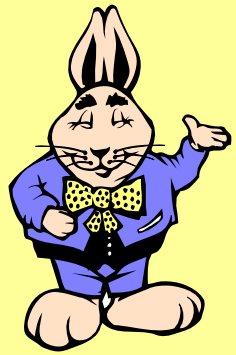
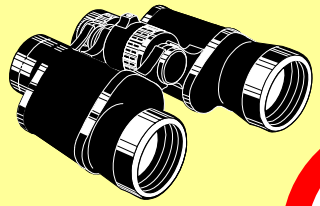
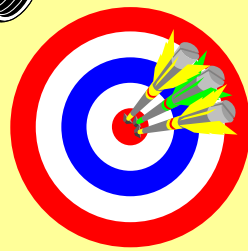
A counting task

What do you want to buy? That'll cost 17p.



The additive composition task

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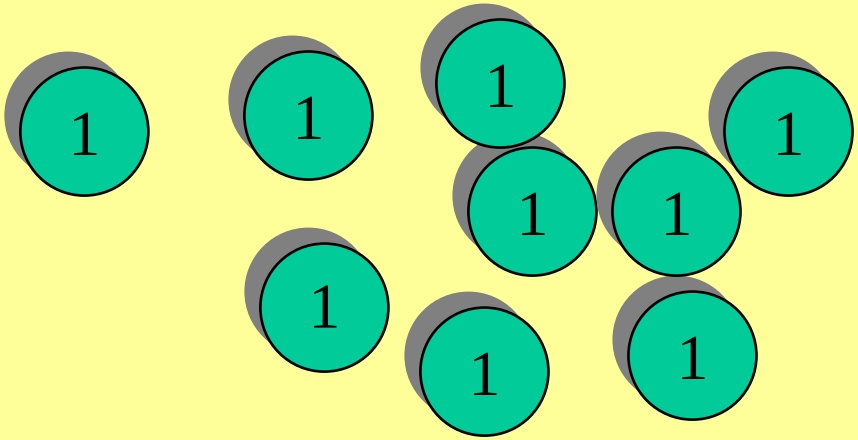
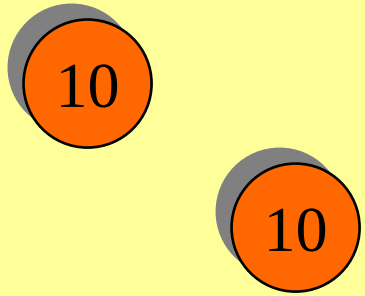




- Although all the children at the end of Year 1 could count 1p coins to 17, only about 60% were able to count the desired amounts using additive composition

Understanding one-to-many correspondence

In each house in this street live 3 dogs. How many dogs live in this street?

66% of the responses to problems of this type were correct by the end of Year 1

We are going to have a party. Every child that comes will get 2 balloons. We have 18 balloons. How many children can we invite?

62% of the responses to problems of this type were correct by the end of Year 1

So, many children are developing an understanding of logical relations and using this to solve mathematical problems. Does that help them learn mathematics in school?

A longitudinal study

- We assessed 59 children at the beginning of year 1 and then looked at their results in mathematics in the standardised school assessment more than one year later
- In order to test whether children's understanding of logico-mathematical relations predict their success in mathematics in school, we need to use a statistical technique that allows us to control for other explanations

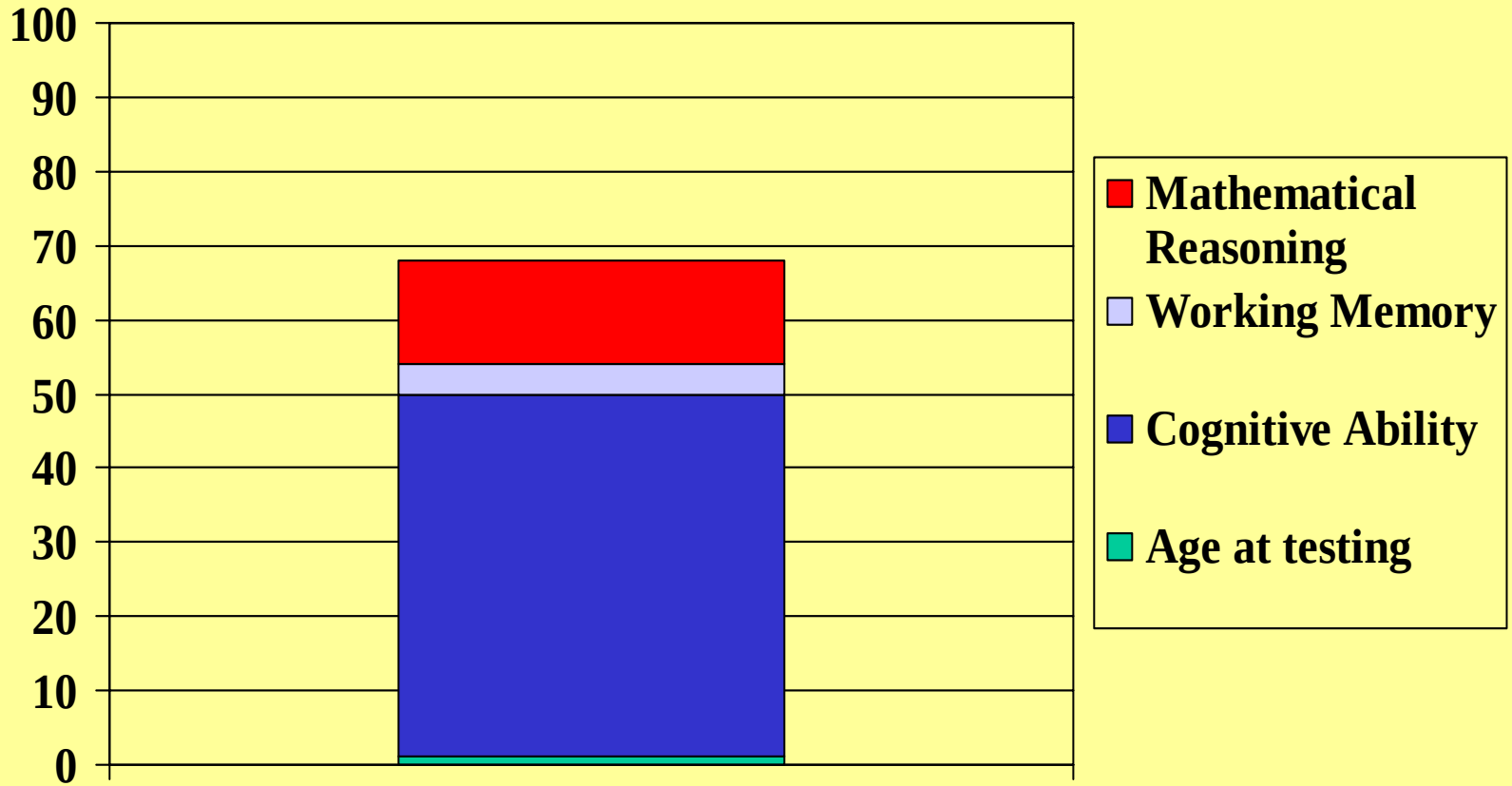
A longitudinal study

- For example: the level of children's understanding is related to age so we need to control for that statistically
- Children's general intelligence also may be related to their logical development so we need to control for that
- Working memory – how well people can remember something while doing something else

A longitudinal study

- These controls are done statistically: in a sense we subtract from the connection between logical reasoning and memory that part of it that is common also to age, intelligence and working memory
- What is left can be seen as the influence of logic on learning by itself – and this is important because it is possible to promote children's logic through teaching

Percent variance explained when predicting mathematics achievement



Conclusions from the longitudinal study

- Logical reasoning does have a specific relation to learning mathematics in school
- The results are important because the children were assessed by the schools, completely independently from our assessments
- They are also important because it is possible to promote children's logical reasoning through problem solving opportunities

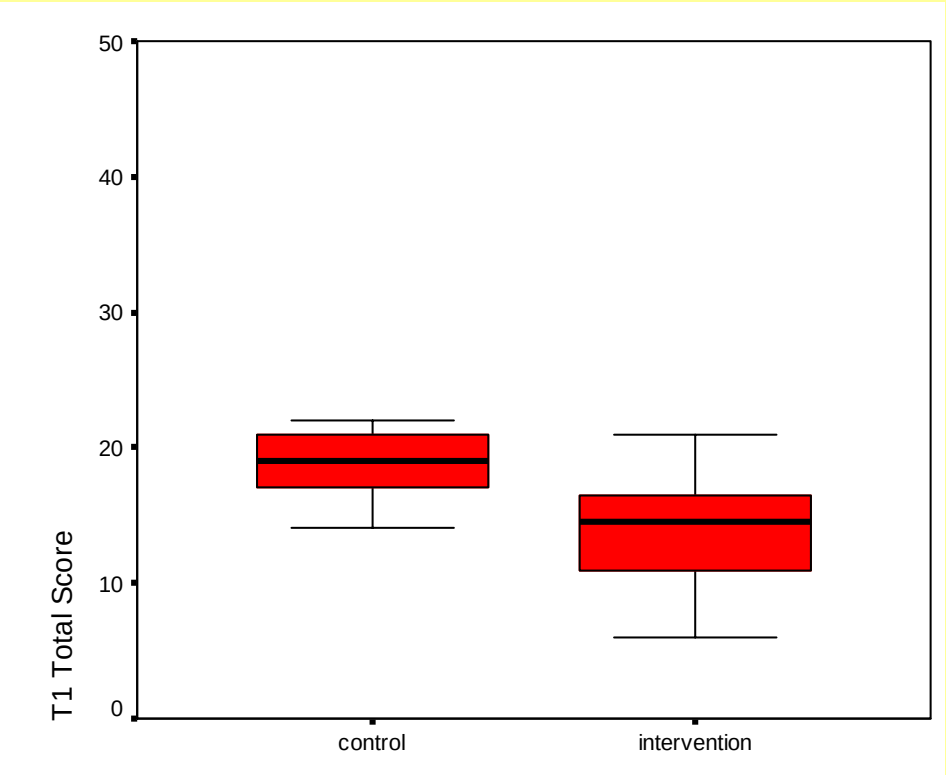
- Children develop much of their understanding of these logical relations informally.
- However, it is necessary:
  - to promote this development in school so that all children are ready for learning;
  - to extend children's reasoning through school instruction so that they connect their informal reasoning with what they learn in school.

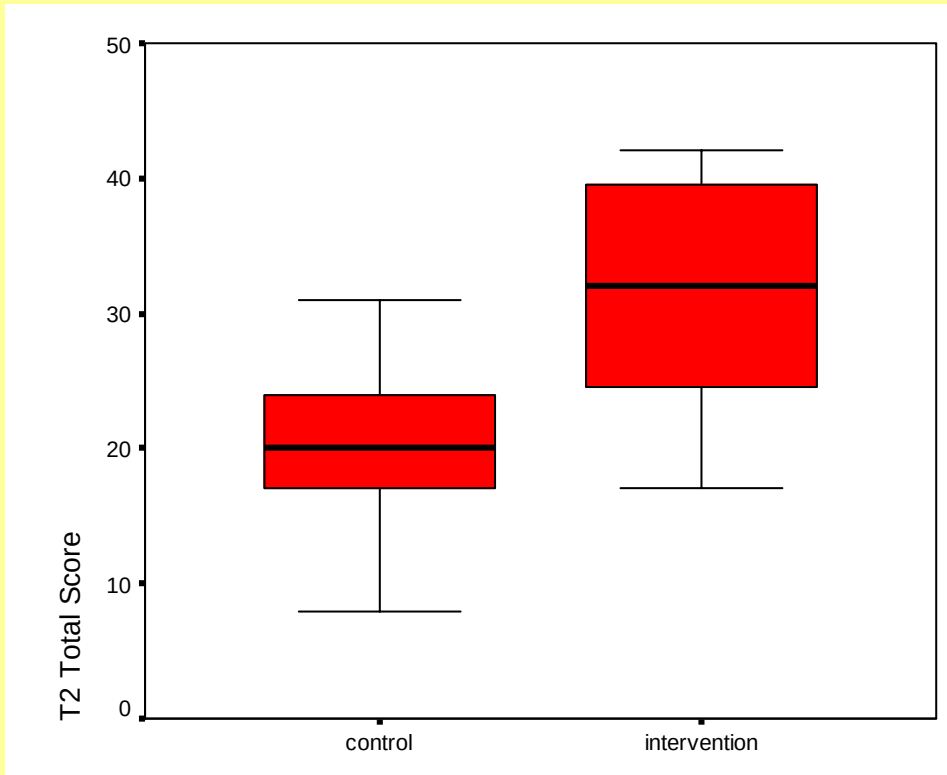
A short-term intervention study

- A small scale study assessed how effective interventions can be
- Children from the same schools in Year 1: Cohort 1 forms the control group and Cohort 2 (sampled in the following year) the intervention group
- Children were selected because they were underperforming within their age group
- Intervention: 12 small group sessions (3 to 5 children in each group) with a researcher
- This was done during the Numeracy Hour (over 12 weeks) so there was no extra teaching time

Working with children on one-to-many correspondences

Results on our logical assessment





In conclusion

- Research on children's reasoning has led to the identification of the logical-mathematical principles that form the basis for children's mathematics learning
- It is possible to promote their understanding of these principles through instruction
- It would be important to extend children's use of these principles with other mathematical symbols

www.ox.ac.uk/edstud/research/child learning