Street mathematics, school mathematics

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Overview

- Why we started the research
  - theoretical assumptions
  - our personal experiences
- The first study: uncovering children’s invisible knowledge
- The second study: a systematic analysis
- Proportional reasoning in and out of school
- The generality of reasoning schemas used in street mathematics

Why we started the research

- Selective school failure: a serious problem in Brazil
- Explanations for poor children's failure at the time: children are behind in their cognitive development due to
  - malnutrition
  - lack of cognitive stimulation

- It was necessary to establish that the children were in fact behind in their cognitive development
- However, most research was confounded because of children’s school experience
- We carried out a study where school experience was controlled for (Nunes Carraher & Schliemann, 1983)
- N=100, 50 from private and 50 from state supported schools; different age, same programme of instruction in school
- No difference in their performance in Piagetian tasks

A helpful theory about culture and cognitive development

- Cultures as providers of tools for thinking and opportunities for practice
- Schools as main providers of tools and concepts
- But our interest in the children led us to notice that they were quicker than us in their everyday commercial activities
- We decided to find out what this context provided


Folha

Questions

- Are the children who succeed in school the same one who have opportunities to practice arithmetic outside school?
- If not, how can they succeed in their street vending and fail in school?
- Could their use of arithmetic in the streets be limited? Could they be just using memory?

The first study: uncovering children's invisible knowledge

- 5 youngsters (4 boys, 1 girl) all from very poor backgrounds who worked in the informal economy
- Age range 9-15 years
- Years in school: <1 (dropped out before end of the year), 3, 3, 4, 8

Design

- We went to the street markets and posed problems to the children that would be part of their everyday activities

  R: How much is the watermelon?

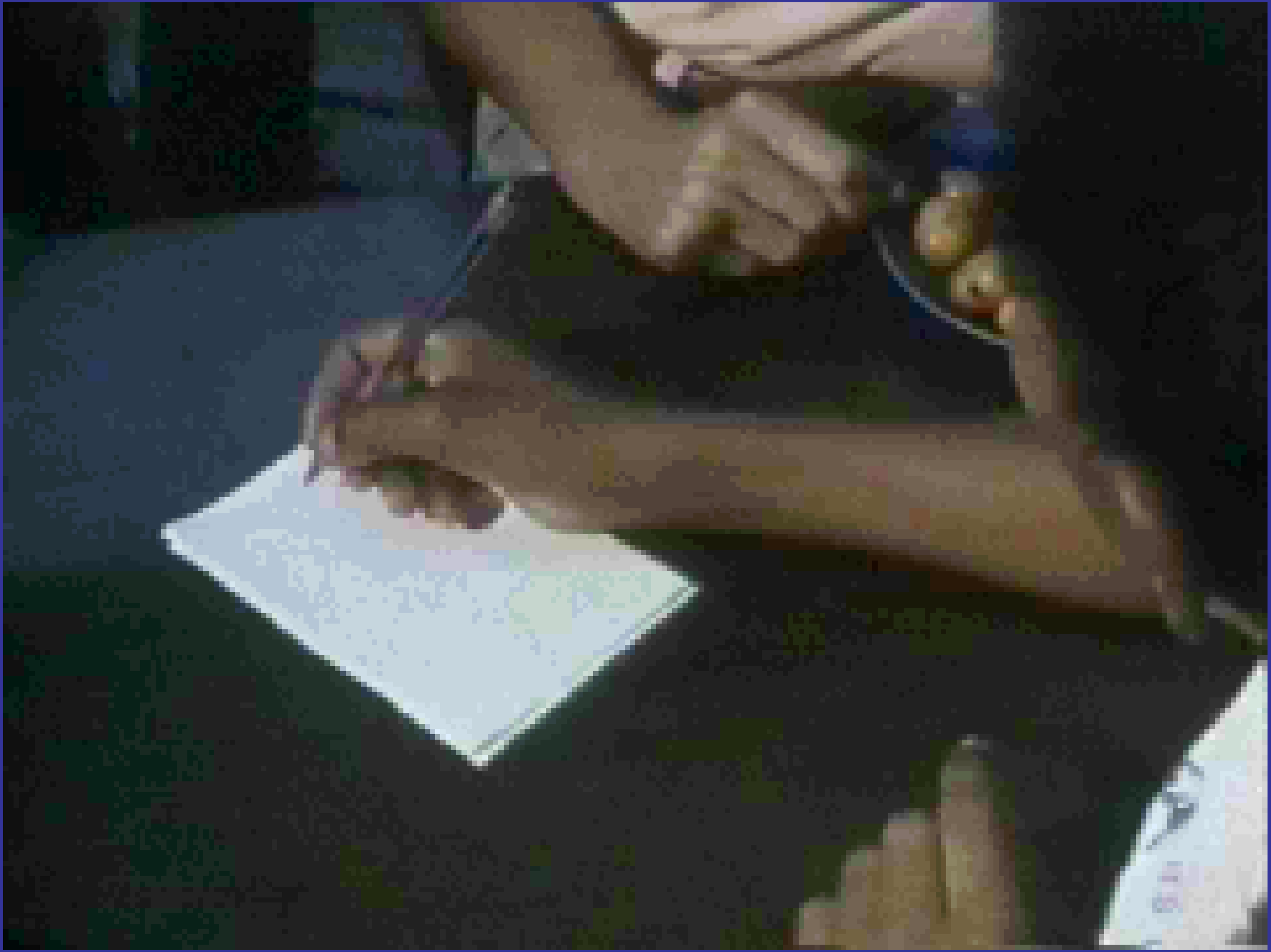
  C: 50 *Cruzeiros* a kilo.

  R: I want 6 kilos.

- The children were invited for a later interview
- Each child solved the same problems as ‘word problems’ or ‘computation exercises’
- 63 problems in the streets, 61 word problems, 38 computation exercises in total

Nunes, Schliemann & Carraher (1993): Mathematics in the streets and in schools

Percentage of correct responses		
In the street market	Word problems	Computation exercises
98	74	37



Many examples of difference in performance

S, age 11, <1 year in school

- Six kilos of watermelon at 50 per kilo

S: 300.

R: How did you do that so fast?

S: Counting one by one. Two kilos, one hundred. Two hundred. Three hundred.

Many examples of difference in performance

Test item: a fisherman caught 50 fish. The second fisherman caught six times the amount of fish that the first one caught. How much fish did the lucky fisherman catch?

S: writes 50

x 6

36

Many examples of difference in performance

Researcher repeats the problem

S: writes 50

$$\begin{array}{r} \underline{\times\ 6} \\ 860 \end{array}$$

Oral answer: 86

Explanation: 6 times 5, 30, carry the 3, and add to 5

Many examples of difference in performance

M, age 12, 8 years in school

4 coconuts, 35 cruzeiros each

M: That will be one hundred and five, plus thirty, that's one thirty five… one coconut is thirty five…that is…one forty.

Many examples of difference in performance

Computation exercise

M: writes 35

x 4

200

Explanation: 4 times 5 is 20, carry the 2, 2 plus 3 is 5, 5 times 4 is 20.

Conclusions

- The children who solved problems successfully in the market were the same that would fail in school
- However, we realised that we did not understand their knowledge
- This made their knowledge invisible

Second study: describing street mathematics

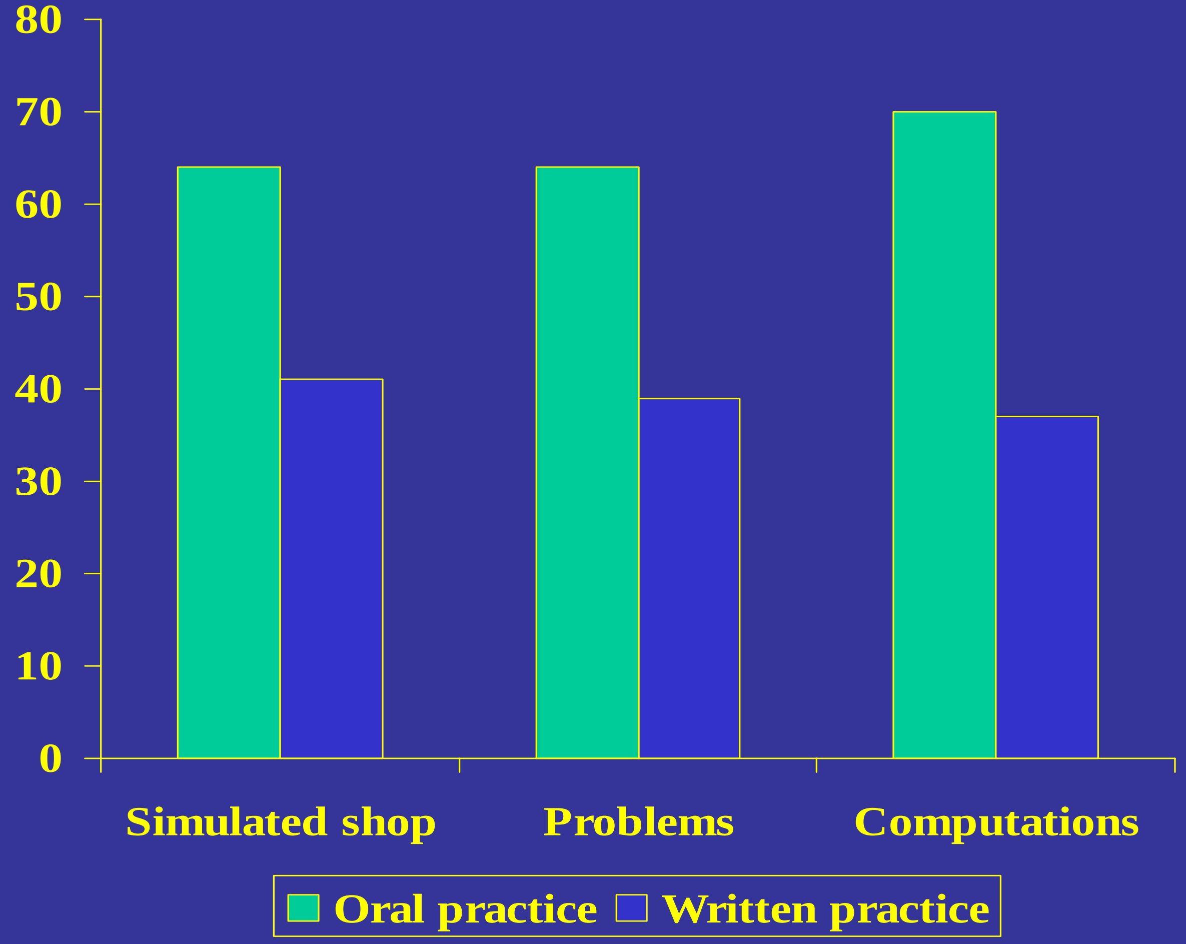
- 16 youngsters in grade 3; age range 8 to 13; mean age: 11.5 (for middle class children, mean age would be between 8 and 9)
- All had received instruction on computation algorithms
- Design
  - Three conditions: simulated shop, word problems and computation exercises
  - Same computations across conditions across participants

- Study carried out in schools in deprived areas
- Same experimenter across conditions, order of condition varied
- Expected to elicit oral and written arithmetic but choice of procedure was open to children
- Analysed when choices were made and rate of correct response by condition and by choice

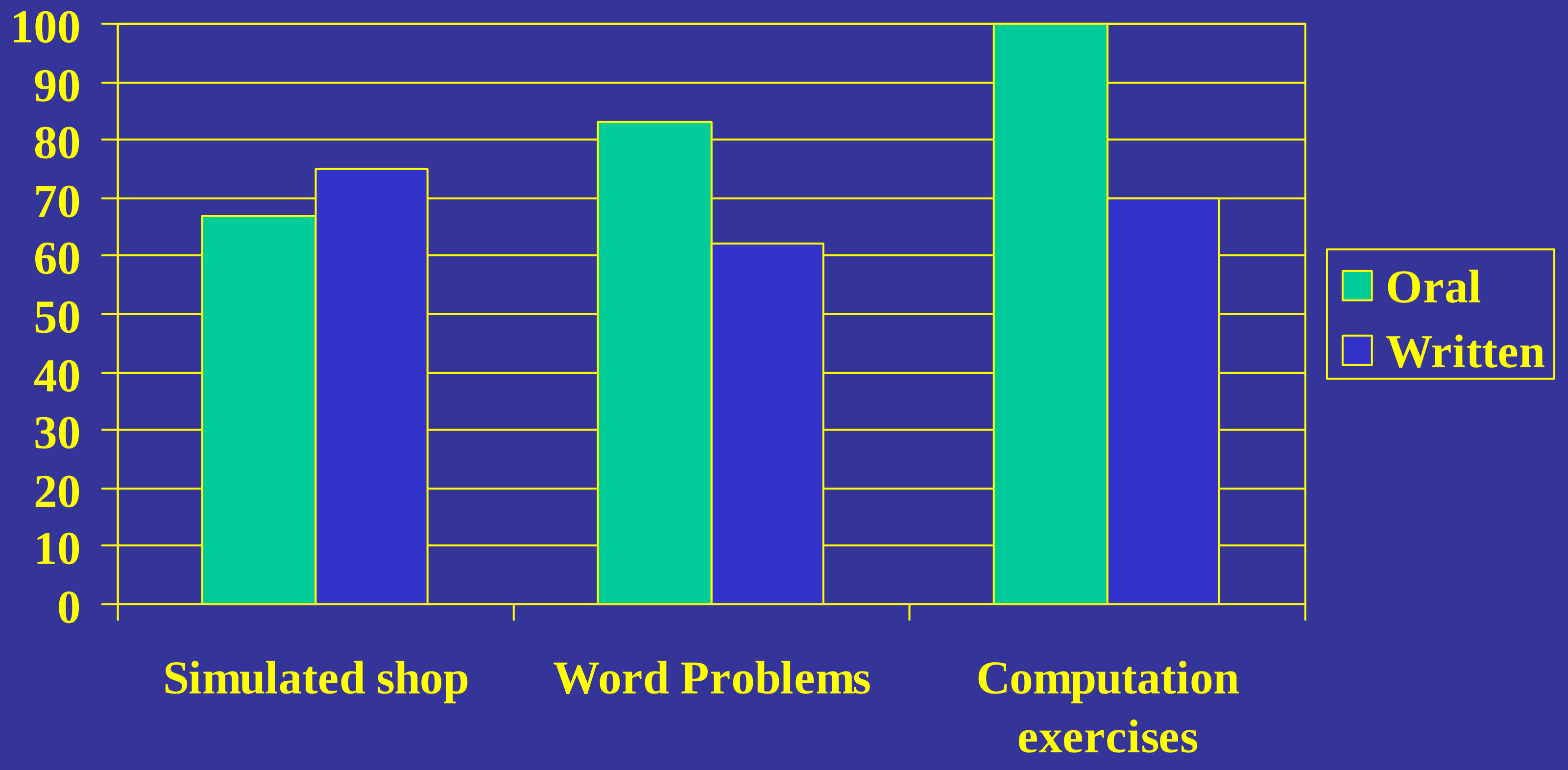
Results

- Choice of procedure:
  - in the simulated shop, above 80% oral procdure for all four operations
  - in the word problems, about half of the additions were done orally but for the other operations over 60% were done orally
  - in the computation exercises, approximately 20% of the divisions were done orally but for the other operations only about 5% were done orally
- Conclusion: condition is a strong influence on choice of procedure

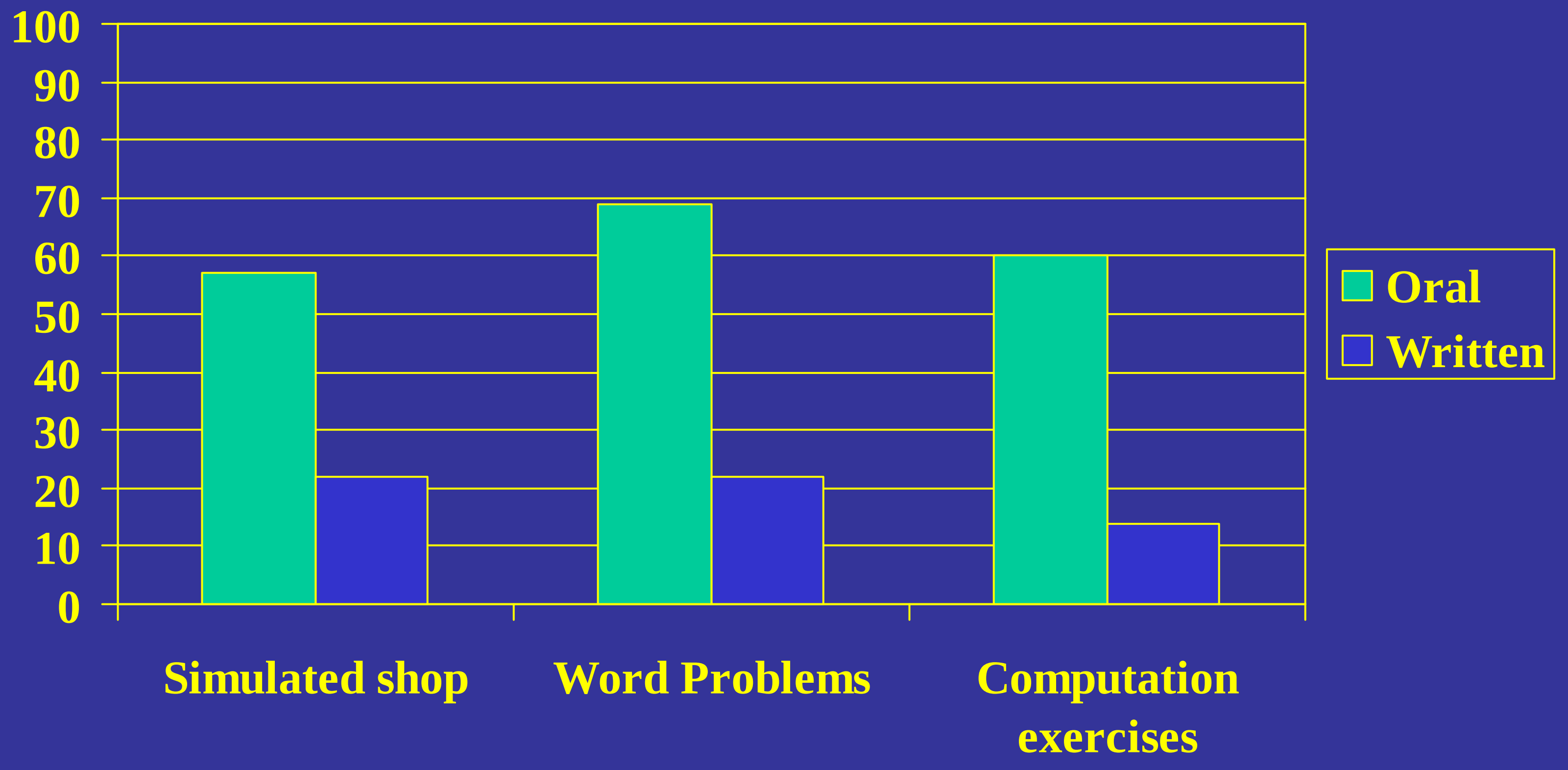
Percentage correct by type of arithmetic practice (Nunes, Schliemann, & Carraher, 1993)



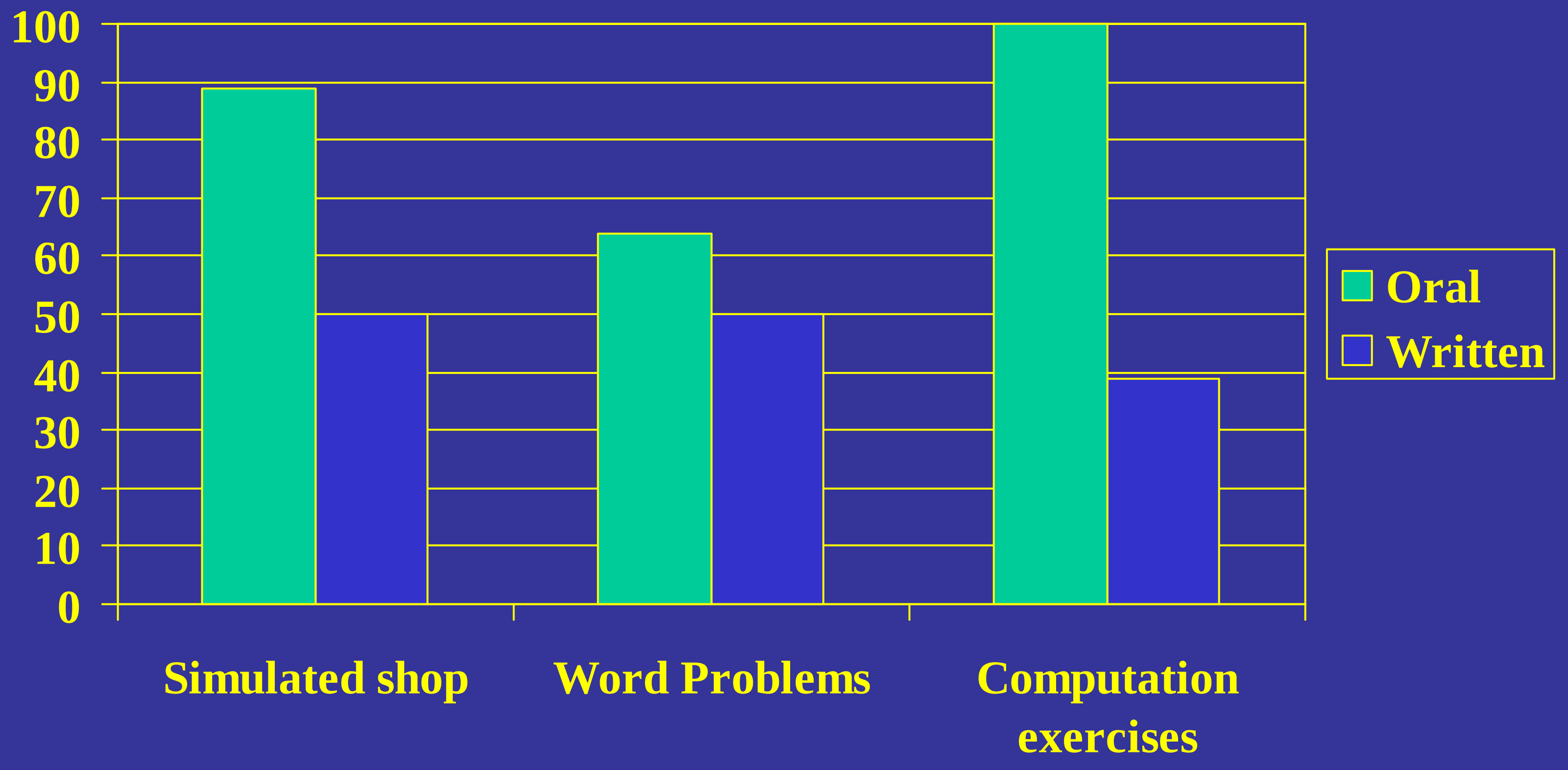
Percent of correct additions by condition and procedure



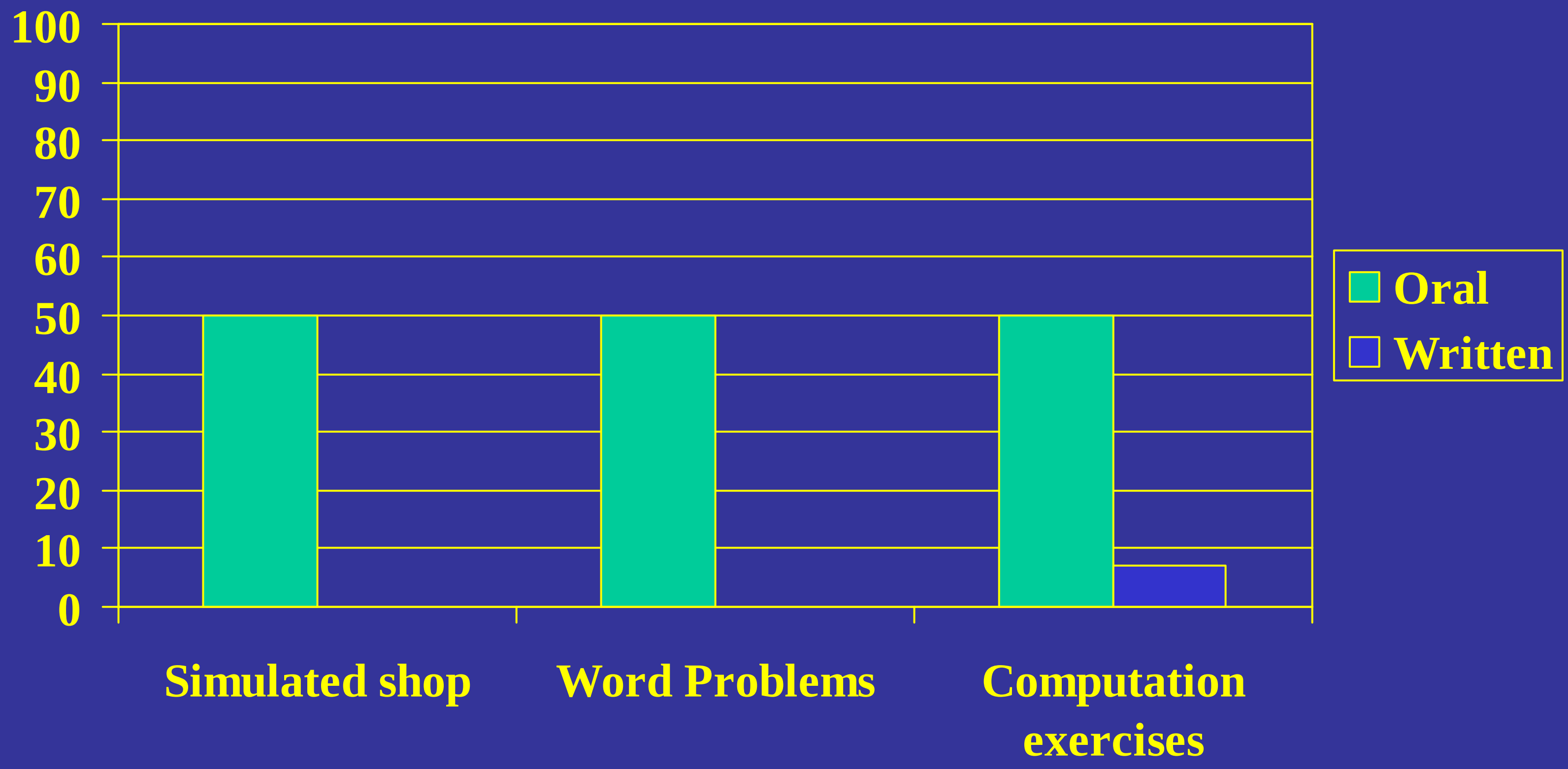
Percent of correct subtractions by condition and procedure



Percent of correct multiplications by condition and procedure



Percent of correct divisions by condition and procedure



Conclusions

- The crucial difference between the conditions seemed to be mediated by the choice of procedure
- In all three conditions and with all four operations, the youngsters were more successful with oral than written procedures

Analysis of the procedures

- Addition and subtraction are based on decomposition

Word problem, 200 – 35

L: It it were 30, then it would be 70. But it is 35. So it’s 65, 165.

Simulated shop: 243-75

You just give me the 200. I’ll give you 25 back. Plus the 43 that you have, the 143, that’s 168.

Analysis of the procedures

Computation exercise: 252 – 57

Take the 52, that's 200, and five to take away, that's 195.

Decomposition uses knowledge of the number system and associativity of addition and subtraction

These are also used in written algorithms but in a different way


Suppose you bought something for 580
and then something for 270,
how much would you have spent?

What are the differences between the procedures?

- Oral arithmetic works through the manipulation of quantities; written arithmetic works through the manipulation of symbols
- Oral arithmetic works from larger to smaller; written arithmetic works the other way
- Within-subject design and thus differences in performance cannot be explained by ‘individual differences’ or ‘group differences’

The logic of oral multiplication

R: I want 10 coconuts (35 *Cruzeiros* each)

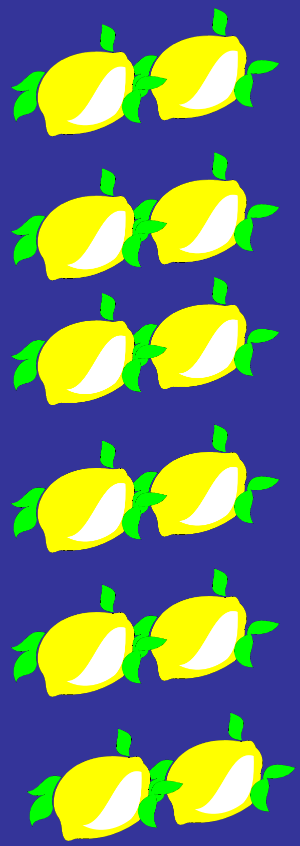
M: 3 would be 105; with 3 more, that will be 210. I need 4 more. That is…315…I think it is 350.

Oral multiplication seems to work through correspondences and scalar computation using ‘easy’ groups.

Counting money, not just bananas

12 lemons, 5 *Cruzeiros* each

MD (separates two lemons at a time as she counts out loud):



10

20

30

40

50

60

The use of correspondence reasoning is very clear in all examples of oral arithmetic; in written arithmetic the procedure involves symbol manipulation

Oral and written division

Word problem: *5 boys got 75 marbles to divide equally among themselves. How many marbles for each one?*

Fabio: *If you give 10 marbles to each, that's 50. There are 25 left over. To distribute to five boys. That's hard. … That's 5 more each. That's 15 each.*

Fabio operates on quantities, not on digits; the use of correspondences is also clear in division

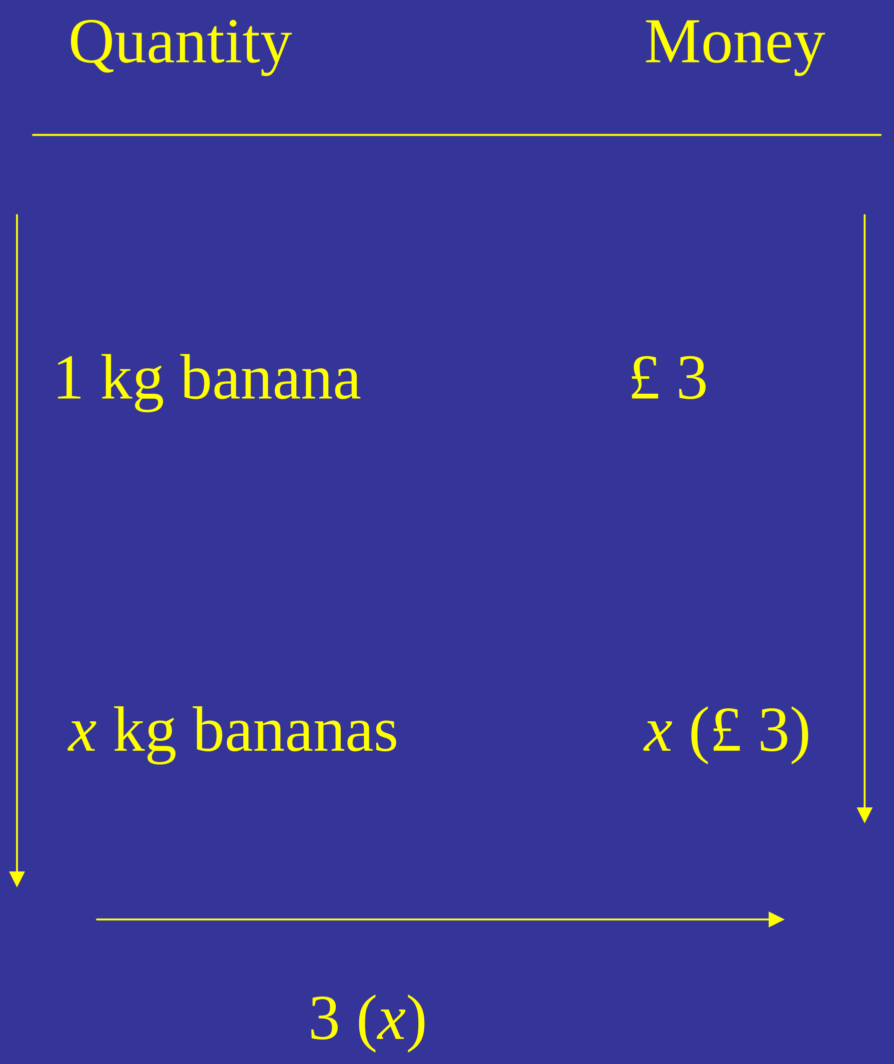
Can street mathematics handle proportions?

- Proportional reasoning involves relations between variables whereas additive reasoning involves putting sets together or separating them
- In Piagetian theory, the scheme of proportionality is a formal operations achievement

Two ways of thinking about relations between variables

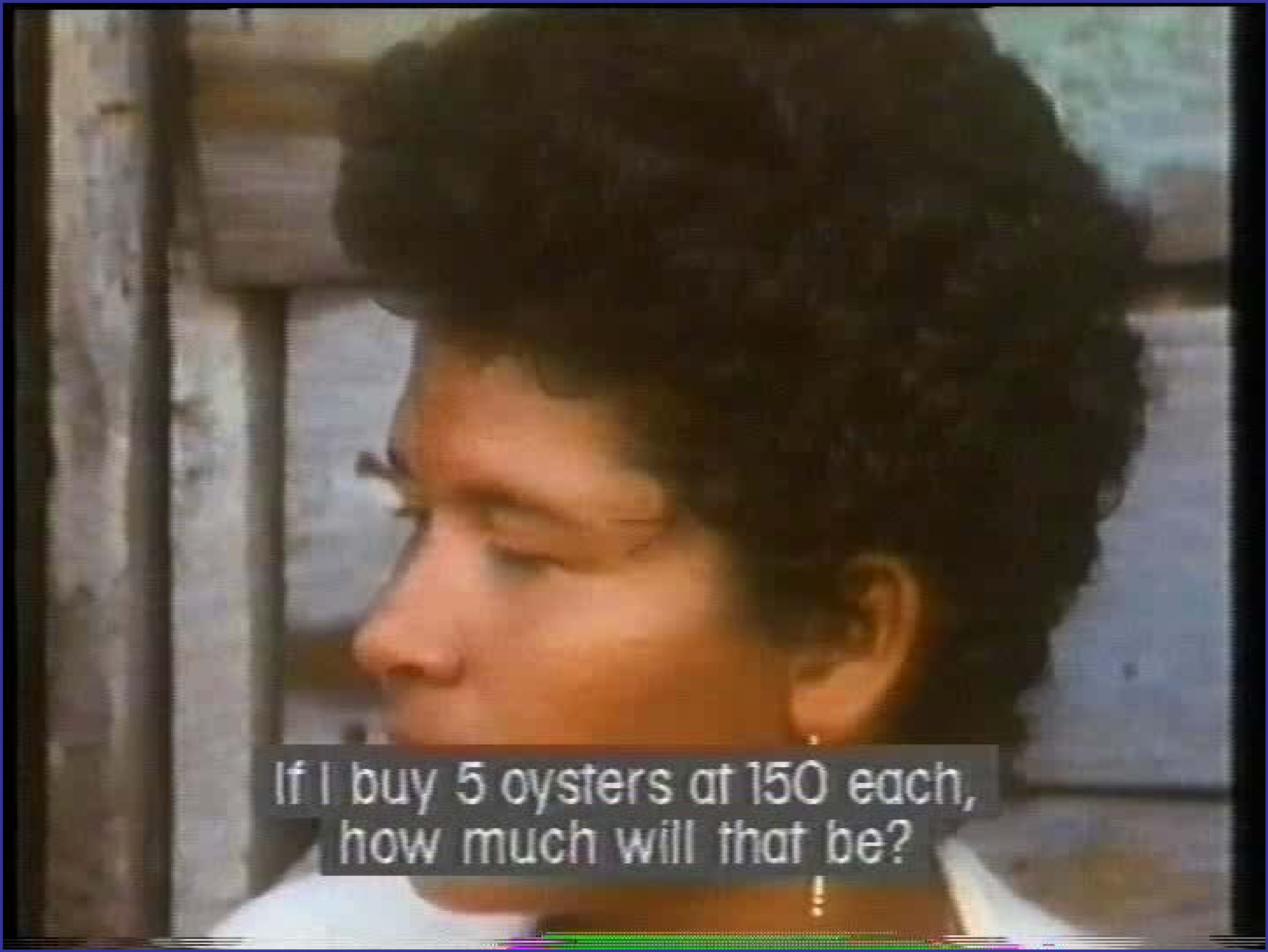
The fixed ratio is maintained if both variables are multiplied by the same number

Any point in one variable is connected to the other by a fixed value



Multiplicative reasoning outside school

- People working in the informal economy have no difficulty in giving the price for larger purchases, even if the price is hypothetical (not the price the person is selling the product for)

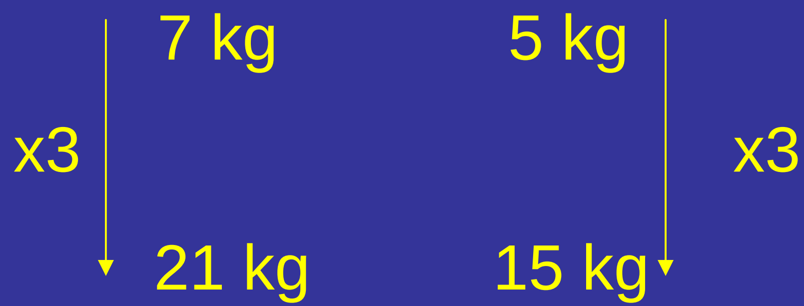

If I buy 5 oysters at 150 each,
how much will that be?

The question of generalisation

- Can people who learned mathematics outside school deal with relations between variables that are not quantity and price?
- Fishermen and foremen

The fishing practice

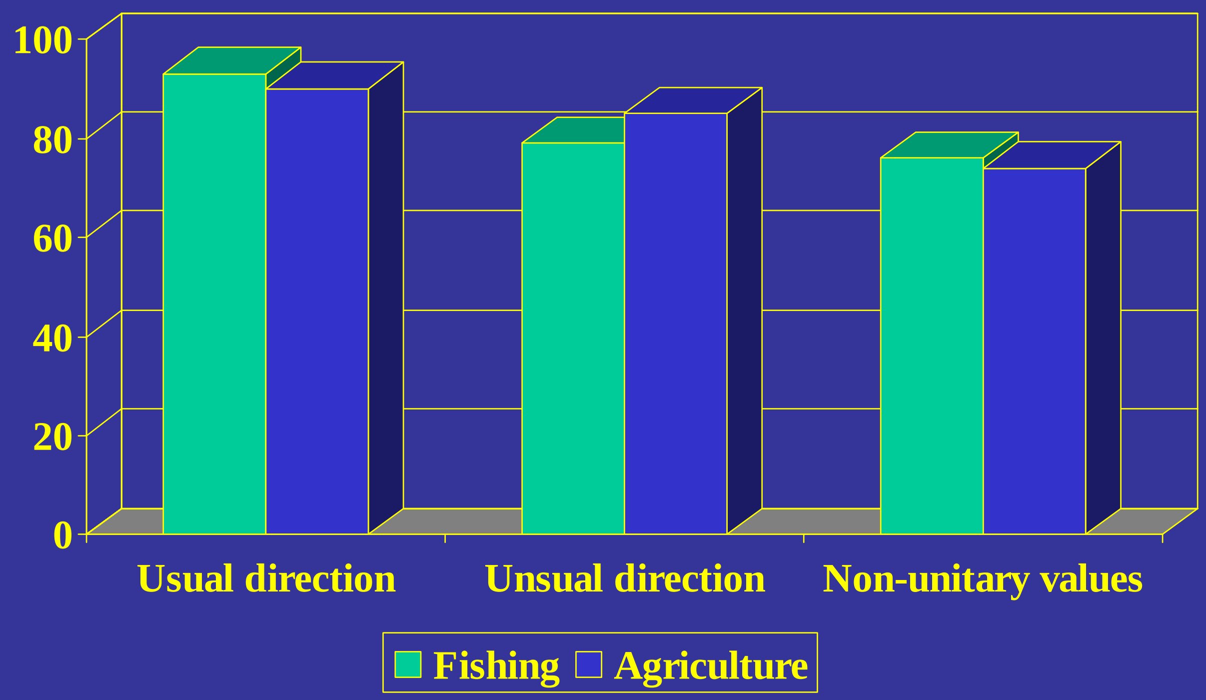
- Fish is sold unprocessed to a middle-man
- Middle-man salts and dries; weight is reduced in processing
- Unprocessed-processed food ratios are known (how much fish will the middle-man sell?)
- Aim of study: assess the use of the scheme of proportionality with unusual values in unusual direction and different problem contents (fishing and agriculture)



The reasoning is so obvious to the fishermen that they do not explain it when the calculation is easy. They only explain it when the calculation is a bit more difficult and they calculate speaking aloud.

- Participants: 19 fishermen; level of schooling from none to grade 5 (proportions are taught in grade 6)
- Procedure: interviews were carried out on the beach; no paper and pencil; interviewers were familiar to the fishermen
- Design: contents fishing and agriculture
  - unprocessed-processed food (usual direction)
  - processed-unprocessed food (unusual direction)
  - unprocessed-processed food, non-unity values

Proportion correct responses by type of question and content



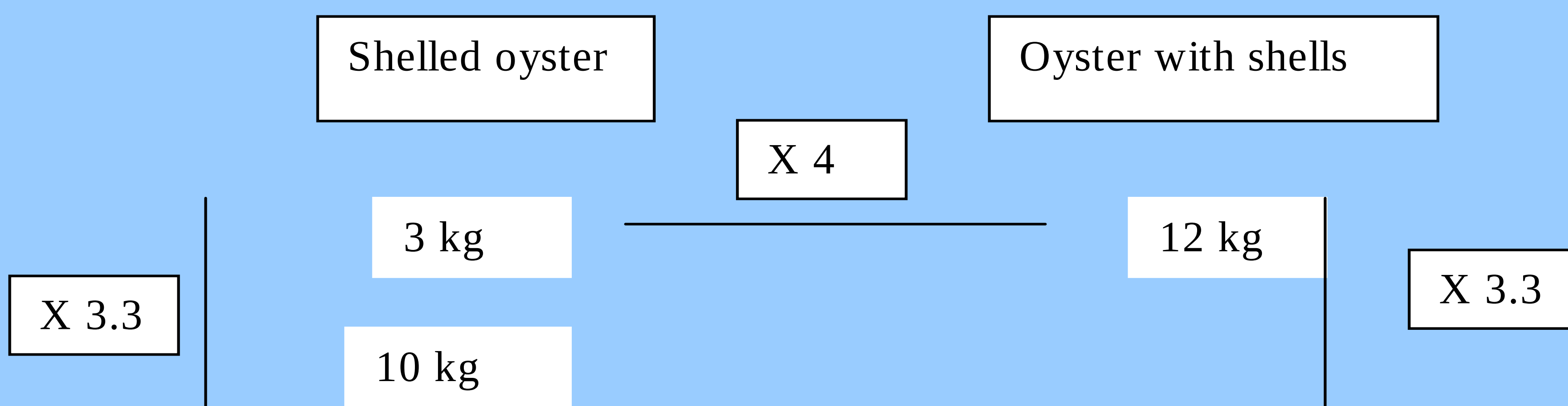
Difference between fishing and agriculture problems was not significant.

<u>When scalar solution is easier to calculate</u>: There is a type of oyster in the south that yields 3 kilos of shelled oyster for every 10 kilos that you catch; how many kilos would you have to catch for a customer who wants 12 kilos of shelled oyster?

	Shelled oyster		Oyster with shells	
		X 3.3		
X 4	3 kg		10 kg	X 4
	12 kg			

83% correct

<u>When functional solution is easier to calculate</u>: There is a type of oyster in the south that yields 3 kilos of shelled oyster for every 12 kilos that you catch; how much do you have to catch if a customer asks you for 10 kilos?



70% correct; diff is significant

fish12 kg; 3 kg shelled; how much do you need to fish for 10 kg shelled?

F: *On average, 40.*

R: *How did you solve this one?*

F: *It's because we make it simpler than using pencil. … It's because 12 kilos give you 3; 36 give 9. Then I add 1 and 4 to give 10.*

(Note that the constant is 4 and that the fisherman knows that 1 kg shelled corresponds to 4 kg fresh. But instead of multiplying 10 x 4 he uses a scalar solution; note the clear use of correspondences)

- There us a kind of shrimp in the south that yields 3 kilos of shelled shrimp for every 18 kilos you catch. If a customer wanted the fisherman to get him 2 kilos of shelled shrimp, how much would the fisherman have to catch?

Fisherman: One and a half kilos [processed] would be nine [unprocessed], it has to be nine because half of eighteen is nine and half of three is one and a half. And a half-kilo [processed] is three kilos [unprocessed]. Then it'd be nine plus three is twelve [unprocessed]; the twelve kilos would give you the two kilos [processed] (p. 112).

3 kg shelled	18 kg
One and a half	nine
Half kilo	three
1.5 + 0.5 = 2	9 + 3 = 12

Why such awkward solutions?

- The focus is on quantities and correspondences
- A scalar solution maintains the focus on quantities: 2 kilos 3 times is 6 kilos; half of 9 kilos is 4 and a half kilos
- The functional solution requires focusing on the relation between the quantities: kilos of fresh shrimp divided by kilos of shelled shrimp is a relation between the quantities
- Is functional reasoning a product of schooling?

A comparison with students

- Students (N=22) were compared with a group of fishermen
- Students had between 9 and 11 years of schooling; the fishermen had between 0 and 9 years, with an average of 3.5 years of schooling

- They were given problems similar to those presented earlier
- Their overall performance did not differ but there were differences in some types of problems
- Most importantly: whereas the fishermen did not perform differently in scalar vs functional problems, the students performed significantly more poorly in the functional problems than the scalar problems
- Schooling had not fully developed their functional reasoning and seems to have interfered with the use of the scalar strategies

Conclusions

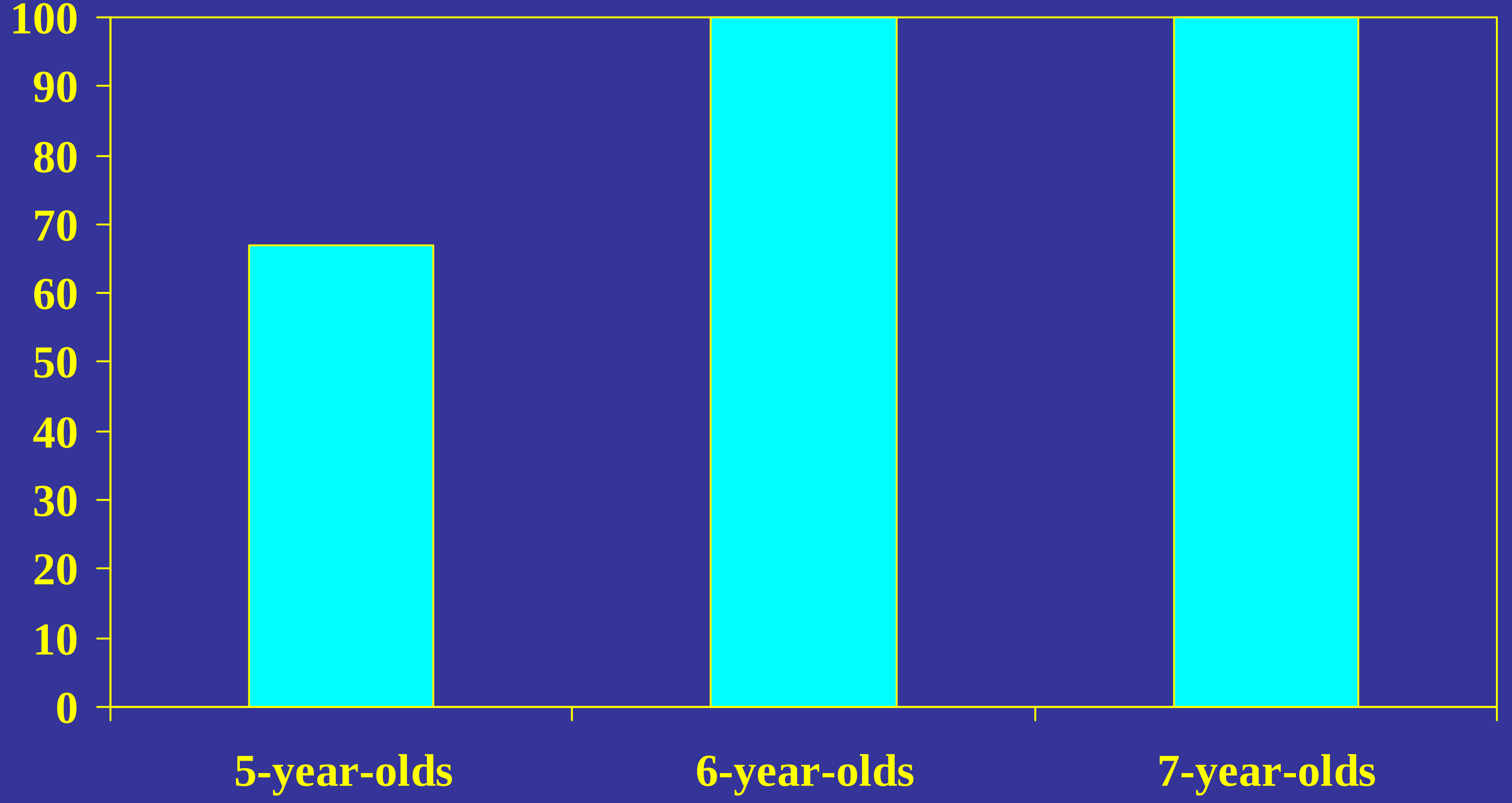
- Fishermen develop a scheme of proportionality
- Its application is not restricted to the usual numbers, usual direction of calculation, and usual content (fishing)
- Strategies have the same characteristic of other oral solutions, with focus on quantities
- Reasoning relies on the schema of one-to-many correspondence
- Status of functional solution needs further investigation

How general is correspondence reasoning?

- When does it develop and what happens to it?
- Our hypothesis is that it develops quite early
- However, schools are currently not promoting the use of correspondence reasoning

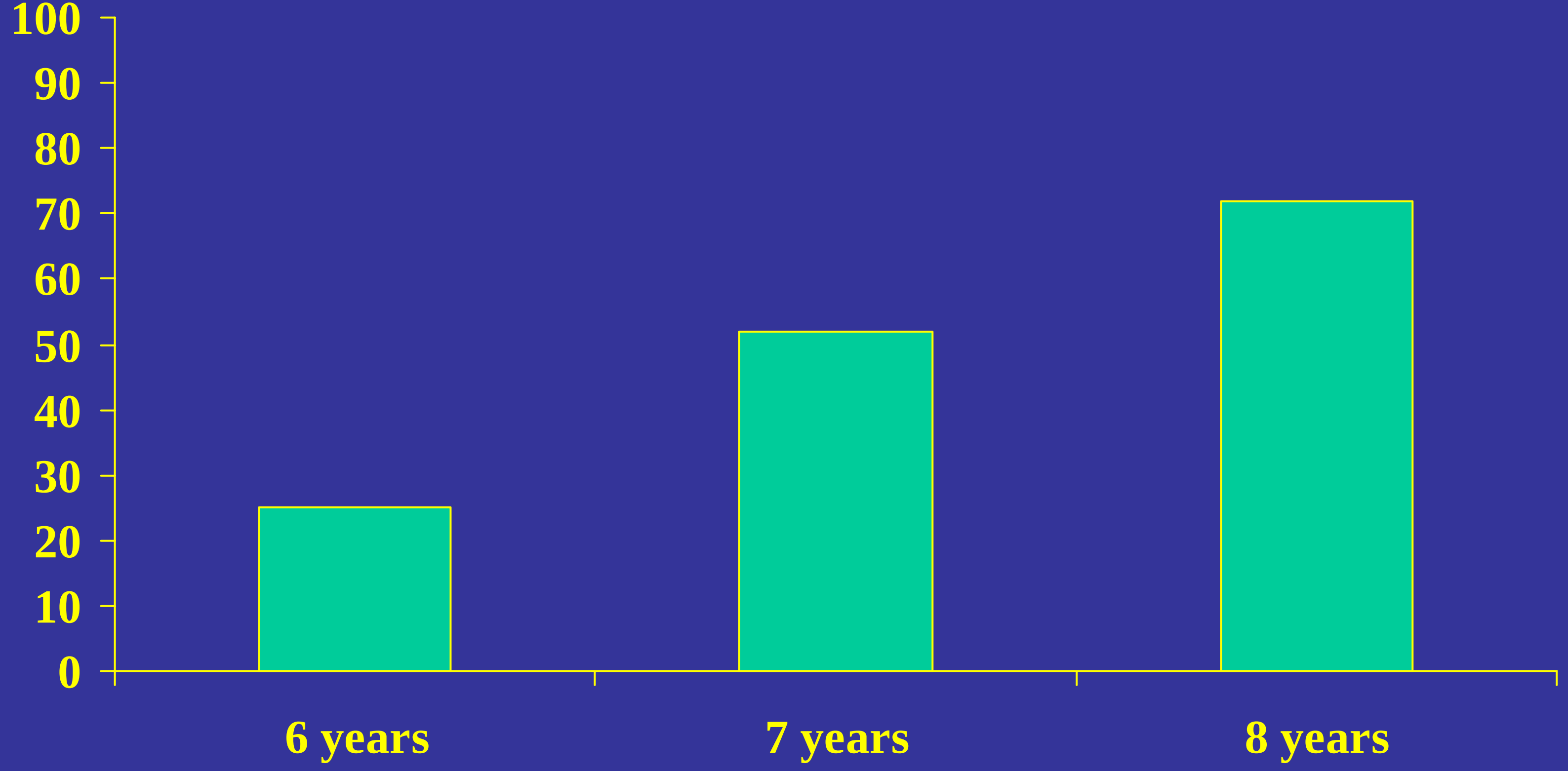
In each house in this street live 3 dogs. How many dogs live in this street?

Percentage of correct responses in the concrete situation



Each child that comes to the party will get 2 balloons. I have 18 balloons. How many children can I invite?

Percentage of correct responses in inverse multiplication problems (Correa, 1994)



Conclusions

- The schema of one-to-many correspondence develops early and children can use it to solve problems
- Schools do not typically take advantage of it to teach multiplication and division
- Research that investigates its use in school is urgently needed

Final conclusions

- The study of street mathematics has helped us see children's invisible knowledge
- Oral arithmetic relies on the same principles used implicitly in written arithmetic
- Street mathematics is not restricted to additive reasoning and includes proportional reasoning

Final conclusions

- Street mathematics has revealed the importance of one-to-many correspondence in multiplicative reasoning
- We have shown that children as young as 5 and 6 years can use this reasoning to solve problems
- We have also shown that teaching children about multiplication using correspondences is more effective than teaching through repeated addition
- Long term research is needed

Congratulations you have reached the end!

Thank you for all your hard work