Summary of the results of teaching children about quantitative problem solving in primary school

The project about which we report here was supported by the Nuffield Foundation and was implemented in two phases. In the first phase, children from three schools in Oxford were randomly assigned to one of three groups:

1. a **problem solving group**, that worked with researchers on problems that did not involve probabilistic reasoning;
2. a **probability group**, that worked with researchers on problems that involved probability concepts and reasoning;
3. an **unseen control group**, that stayed in the classroom with the teacher.

These groups were created to test whether:

- both groups of children who had this extra opportunity to work on mathematical problems would develop general problem solving skills such as reading problems carefully, thinking before trying to calculate, and using diagrams to solve problems;
- each of the groups who worked with the researchers would develop specific knowledge about mathematical concepts: the problem solving group would think about how to connect problem situations with operations and the probability group would learn to think about randomness, sample space, and quantification of probability.

In this first phase of the project, we found that:

- the **problem solving** and the **probability groups** made more progress in problem solving than the unseen group from pre- to post-test; the **unseen control group** did not make significant progress from pre- to post-test in problem solving;
- the **probability group** made more progress than both other groups in probability questions.

Our analysis of the mathematical reasoning involved in solving applied problems suggests that children must learn:

1. to **represent quantities with numbers and manipulate the numbers** to find answers to problems; these manipulations involve **necessary relations between numbers and operations** (e.g. the inverse relation between addition and subtraction is necessary);
2. to **reason about relations between quantities** in order to know which operations to use to solve a problem; relations between quantities are **defined in the context of a problem**.

In the second phase of the project, the materials tried out in the first phase were used by teachers with their students. A total of 25 schools and approximately 810 students participated. The results summarised here do not include the data from three classes that did not complete the project. The teachers who tried out the programme received professional development in one of three ways: either a full day workshop held at the University of Oxford, or a twilight session held at a school that works with a cluster of schools in the same area, or a one-to-one instruction with a researcher held at the teachers school. We found that the mode of teacher preparation did not affect the results, presumably because they all were using the same problems for discussion in the classroom, so no further analysis of this variation is presented here.

The teaching programme focused on learning to think about necessary relations in part 1 and learning to identify contextual relations between quantities in part 2. The inverse relation between addition and subtraction and between multiplication and division was explored in part 1. In part 2, the students learned to use diagrams to represent quantities and the relations between them.

We analysed the results with respect to three measures. The first was based on children's use of the inverse relation between addition and subtraction with whole numbers in a game, the pinball. The second was based on the students use of this same reasoning in equations, a type of problem not used during the teaching. The third measure was a general problem solving measure, which included all the problems in the pre- and post-tests. Each of the three graphs shows the results for each of these measures.

In each of the comparisons, both groups made significant progress from the pre- to the post-test, and the Problem Solving Group made significantly more progress than the Probability Group in the problem solving measures (note that the reverse was the case in the probability measures).

Separate analyses were carried out for the group whose scores in the pre-test was below the median and for the group whose scores were above the median (see Figure 1). The analyses showed that students who started the programme with results below the median made significantly more progress from pre- to post-test in general problem solving and in the pinball problems than a comparable group of students taught about probability. However, this was not the case in the equations measure. Those who had scores in the top half at pre-test made significantly more progress in general problem solving and in the equations measures than the comparable group taught about probability, but not in the Pinball problems. Thus, students in the weaker group benefited more in less difficult problems and those in the stronger group benefited more in more difficult problems, but both groups made
significantly more progress in the overall measure of problem solving than the comparison group that was taught about probability.

![Figure 1. Means and standard error of the means by group in the different problem solving measures.](image)

We also found that boys performed better than girls on the pre-test so we analysed the results by group and gender. We wanted to know whether the programme was more effective for boys or girls or whether it was equally effective for both groups. It was possible for the programme to be equally effective for both but the gap between boys and girls in problem solving not to change.

![Figure 2. Means for boys and girls by group at the pre- and post-test overall measure of problem solving](image)

The statistical analyses showed that the boys performed significantly better than the girls at pre-test. At post-test, both boys and girls in the Problem Solving group performed significantly better than the boys and girls in the Probability Group. As the parallel lines in each graph indicate, the effect of the programme did not differ for boys and girls: boys and girls in each group show similar levels of improvement from pre- to post-test. This means that the programme did not close the gap between boys and girls but that the girls profited as much as the boys from it.

In summary, the teachers were able to successfully implement the Problem Solving programme and the pupils who participated in the programme performed significantly better than the Probability Group in the problems that do not involve probability. The Probability Group progressed significantly in the problem solving assessment from the pre- to the post-test, but not as much as the group that was specifically taught about problem solving. We conclude that this project provides crucial evidence that it is possible for teachers to teach problem solving in meaningful ways in primary school.