UNIVERSITY OF OXFORD

Developing language-responsive mathematics classrooms

FINAL REPORT

APRIL 2025

UNIVERSITY OF OXFORD

Developing language-responsive mathematics classrooms

JENNI INGRAM, ELIZABETH KIMBER, ROBERT WARD-PENNY, GABRIEL LEE, KIRSTIN ERATH, NÚRIA PLANAS AND JILL ADLER

FINAL REPORT

APRIL 2025

Publication details:

First published in 2025 by the Department of Education,
University of Oxford, 15 Norham Gardens, Oxford, OX2 6PY

www.education.ox.ac.uk

ISBN – 978-1-7384754-2-1

Contents

UNIVERSITY OF OXFORD

Acknowledgements

This report summarises the Nuffield Foundation funded project “Developing language-responsive mathematics classrooms”.

The authors thank the teachers who worked with us to develop the materials arising from the project. They not only contributed to the guidance materials, they also tested the resources in their own classrooms and allowed us to observe and video their teaching. Without their collaboration, this project would not have been possible.

The authors would also like to thank the Association of Teachers of Mathematics, Colin Foster, Dave Hewitt, Dietmar Küchemann, National Centre for Excellence in the Teaching of Mathematics, and NRICH for allowing us to use the mathematical tasks they developed in the project.

Finally, the authors thank the members of the Advisory Board for their invaluable advice and feedback throughout the project.

The Nuffield Foundation is an independent charitable trust with a mission to advance social well-being. It funds research that informs social policy, primarily in Education, Welfare and Justice. The Nuffield Foundation is the founder and co-funder of the Nuffield Council on Bioethics, the Ada Lovelace Institute and the Nuffield Family Justice Observatory. The Foundation has funded this project, but the views expressed are those of the authors and not necessarily the Foundation. Visit www.nuffieldfoundation.org

Nuffield Foundation

Executive summary

Aims and background

Decades of research have shown the importance of teaching that supports students to communicate mathematics and to communicate in mathematical ways. Primarily, this research shows that the linguistic aspects of learning mathematics need to be amplified rather than simplified and that language and mathematics need to be supported and developed at the same time.

Language is more than the vocabulary or words used; it includes aspects such as the specific grammar of mathematical arguments, as well as what counts as a mathematical explanation or justification. Language is the means for communicating in mathematical ways.

The language involved in learning mathematics is both distinctive and challenging. Word meanings are often more precise and can differ from their use in everyday life. Mathematical communication also frequently involves dense phrases, nominalisations, and logical connectors used in specifically defined ways. Diagrammatic and graphical representations have an underlying mathematical structure that makes them distinct from illustrations. Mathematical arguments need to be logical, precise and succinct.

All students encounter challenges to do with the linguistic demands of learning mathematics. These challenges become more limiting if there are restricted opportunities to learn mathematics in language-responsive ways. This concern is further heightened for those whose opportunities have been constrained by disadvantage. Managing these challenges by simplifying the language limits students' opportunities to learn mathematics, whereas amplifying the language can promote familiarity and fluency.

The Developing Language Responsive Mathematics Classrooms project researched the impact on teachers' classroom practice and students' mathematical communication of using language-responsive design principles and teacher moves.

These design principles include:

Engage students in rich discourse practices
These practices include students explaining their reasoning or the reasoning of others, justifying solutions and developing mathematical arguments.

Establish a variety of mathematics language practices and routines
These practices include teaching strategies such as think-pair-share, as well as clearly establishing what counts as a mathematical explanation or argument and routines of listening to and building on each other's ideas.

Connect different representations and language varieties
Learning mathematics involves making connections between different representations, such as diagrams, graphs, symbols and spoken descriptions and moving between them. It also means making connections between more informal or everyday ways of talking about or explaining mathematical ideas or processes and the more technical or precise mathematics ways of talking.

Draw on students' own linguistic resources
Students will have their own ways of communicating, which may involve other languages or discourses. Language-responsive teaching builds on and uses these resources that students bring with them to the classroom.

Combine language and mathematics learning opportunities

Language and mathematics are intertwined and, therefore, need to be integrated into the teaching and learning of mathematics. Key terminology is learnt through opportunities to use it in meaningful tasks and contexts.

Develop students' awareness of the roles and forms of language in mathematics

Language needs to be amplified, not simplified. Being explicit about word choice or the structures of mathematical arguments can support students to both learn how to communicate in mathematical ways and build meanings for the complex and abstract mathematical concepts they are learning.

These principles rely on the use of several teacher moves (deliberate and intentional classroom strategies or actions) that have been established by research as supporting students' learning in mathematics. These teacher moves include:

1. Planning and preparing for discussions that focus on mathematical concepts
2. Understanding and connecting students' ideas and mathematics, making it as accessible to as many students as possible
3. Drawing attention to the language practices involved in learning mathematics
4. Encouraging students to participate in cognitively demanding discourse about mathematics concepts and processes
5. Attending to the feedback and evaluation of students' mathematics
6. Purposefully using pauses and silence.

One aim of the project was to improve the classroom experiences of students as they face linguistic challenges when learning mathematics. The project achieved this by developing professional development materials with teachers that focused on three key topics in the Key Stage 3 curriculum: linear equations, angles in parallel lines, and introduction to probability. The professional development materials included task guidance for widely used, familiar classroom tasks and guidance on key concepts and the different aspects of language-responsive teaching. These materials highlighted the linguistic challenges within each topic, focusing on the core mathematical concepts, and opportunities to use the language-responsive design principles and teacher moves.

Methods

The project focused on two research questions:

1. How can mathematics teachers support students facing linguistic challenges when thinking about and working on mathematically demanding tasks?
2. How can mathematics teachers support students to engage in mathematical classroom interactions around these demanding tasks?

The project took place between February 2023 and July 2024. It involved 7 teachers from 5 schools who worked collaboratively with the project team to design, trial and develop the materials through five design cycles. The teachers and the project team met virtually twice a term for shorter twilight meetings and once a term face-to-face (or hybrid where needed) for a full day professional development (PD) meeting. The teachers and project team also video-recorded some of the lessons where the teachers trialled the developing materials. These meetings and the analysis of the videos led to the refinement of the materials across each design cycle.

The outcomes of this project include three sets of professional development materials and evidence of the impact of these materials on classroom activity, including students' reasoning and students' end-of-topic assessments.

UNIVERSITY OF OXFORD

Summary of findings

This project highlights the important role of language in the teaching and learning of mathematics, with its communicative and epistemic functions.

On the whole, teachers' discussions in the meetings focused on the linguistic demands of the specific tasks they used and also the more general linguistic demands of the underlying mathematical concepts within these tasks. There were also discussions around teacher moves, but these usually arose in the discussions about the tasks. The focus on familiar tasks proved to be a highly successful feature of the project. This helped make the language and conceptual demands more tangible.

Lesson videos

Of the three topics, the highest number of lessons taught and videos recorded were focused on linear equations. This was the first topic area discussed in PD sessions and was returned to in later sessions as part of the design cycles. The fewest lessons taught and videos recorded were focused on probability. This topic area was the last topic discussed in the PD sessions and was only involved in one design cycle. All lesson videos were analysed using a three-stage process; identifying topic related episodes; using a published observation framework where each component related to the quality of mathematics and the nature of discourse was rated on a scale from 1 to 4; and finally using a purposely developed research-based framework focusing on teacher and student moves.

In the videoed lessons, the episodes that included the tasks discussed and worked on in the PD meetings had higher average ratings for all dimensions of classroom discourse, cognitive engagement, responsiveness and quality of subject matter than the other lesson episodes. In particular, the average ratings for the components *Nature of Discourse*, *Engagement in Cognitively Demanding Subject Matter*, *Multiple Approaches to and Perspectives on Reasoning*, *Eliciting Student Thinking*, and *Teacher Feedback* were all over 1 scale point higher in the episodes including project tasks than those that did not include these tasks.

The majority of the tasks that the teachers chose to use during the lessons that were video recorded focused on eliciting students' reasoning and were designed to provide opportunities for students to explain why, rather than explaining how or that. This partially explains the higher average ratings for the Nature of Discourse, Eliciting Student Thinking and Engagement in Cognitively Demanding Subject Matter which measure different aspects of students' reasoning.

In the episodes where at least some of the students' discourse included detailed contributions there were a high number of student moves, with almost as many student moves observed (424) as teacher moves (489) in total. There was considerable variation across the different teachers in the teacher moves and student moves observed. Across all the videos, in these episodes, the second most frequently coded type of student move was students explaining reasons (explaining why), often offering linguistically demanding comments which illustrated their mathematical thinking.

There was also variation in the relationships between the teacher moves and student moves. This variation was not unexpected, but there is little research that considers the sequential nature of these moves. Following a teacher move that focused on enhancing language practices for learning mathematics, there was usually a short answer from students, such as giving a keyword or technical term, but this was often followed by a teacher move that made connections between this short answer and the mathematical ideas in focus. This suggests that in this project, it was common for teachers to follow up a language-focused move with a mathematics-focused move.

The most common follow-up to teachers encouraging students to explain, argue or give reasons was students explaining their reasoning. This is not unexpected, but it speaks to the influence of teacher moves, as well as students' willingness and capabilities if they are explicitly asked to engage in demanding discourse practices. In the videoed lessons, students were encouraged to explain, argue or give reasons both during the episodes focused on tasks from the project and during the rest of the lesson more often than has been seen in other research.

There were also examples of the specific teacher moves considered in the meetings in the episodes that did not focus on the tasks considered in the project, suggesting that there is some transference of these moves into the teachers' wider practice.

Teacher evaluations

Teachers reported that the professional development materials were worthwhile. Several commented on their familiarity with some of the practices discussed in the PD sessions, although the teachers noted that while these practices might be familiar, they were not always embedded in their current practice.

Their evaluations largely focused on two aspects: becoming more conscious of their own language use and connecting this with shifts in students' expectations of mathematics classroom language; and the benefits of lessons involving richer and deeper classroom discussions.

Teachers related their own use of the mathematics register to their being more conscious about the phrasings they used themselves, for example, reassessing multiple phrasings and phrasings students might use. Teachers also connected this awareness with the mathematics.

They compared recent lessons with those from the start of the year, noting that '*we don't often get stuck for the word that we need to use*' which they attributed to students having become used to using the most meaningful words, including in student-student interactions where they might previously have used more colloquial language.

Teachers also reported changes to richer and deeper classroom discussions, which some related to changing expectations of students, including those identified in schools as '*lower ability*'. Teachers commented on a change in their expectations of students in classroom discussion, noting that students were taking arguments further. Some teachers also commented on the affordances of intervening less, to give students opportunities to give explanations and share their reasoning, and affordances of listening more to students.

Teachers reported that they were both making more connections between topics now and that this had a higher priority in their teaching.

One theme that occurred repeatedly in the teachers' evaluations in relation to other points noted above was a shift from focusing on completing tasks, which they called a product perspective, to the process of working on the tasks.

Not all teachers were involved in all the meetings or PD design cycles, usually due to the heavy demands associated with being a school teacher. One of the most challenging aspects of this project was finding times when the teachers could come together, particularly for the face-to-face meetings given the geographical spread of the teachers involved.

Professional development materials

As a result of the project, a range of professional development materials were developed. Overarching these materials are a set of resources describing the principles of language-responsive mathematics teaching, including the research behind them and support tools for analysing classroom practice that focuses on the development of these practices and teacher decision-making behind these practices. The main materials are three sets of topic-specific guidance, focused on linear equations, angles in parallel lines, and probability. Each set includes:

Task guidance

This guidance is for widely used classroom tasks and focuses on the linguistic demands, challenges and opportunities within these tasks.

Key concept guidance

This guidance includes an outline of research into the teaching and learning of key concepts involved in the topic, focused on the linguistic demands and challenges.

Recommendations for further research

This project was a relatively small development project resulting in the production of easy-to-use materials to support the development of language-responsive mathematics teaching. These materials have their basis in existing research, but much of this research has been conducted in other countries or with younger children.

Further research is needed to examine the impact of these design principles and teacher moves on the structure and nature of students' reasoning. While this project and other research shows that the amount of student reasoning in lessons increases with language-responsive mathematics teaching, the nature of this reasoning has not been examined. As part of this project there was a focus on considering structures of explanations and arguments and not just vocabulary, in terms of teachers' awareness of these structures, but also in terms of the quality of students' explanations and arguments. While there were several examples of high-quality student explanations and arguments in the video data and assessment data, further research is needed into what supports students in developing these.

This project also focused on three key topics within the Key Stage 3 mathematics curriculum. These topics were chosen because they involve underlying concepts that are fundamental to understanding mathematics and they involve contrasting styles of argument structure and use of representations. While some aspects of language-responsive mathematics teaching carry across different topics, meaning-related language, some argument structures and meaning-related representations are specific to the mathematical concept or process. Further research is needed to identify these aspects of classroom discourse for other topics in the secondary curriculum.

Suggestions for further implementation

The professional development materials developed address the need for mathematics-focused approaches to professional development as wider research has shown that effective practices are often at the level of a curriculum topic. With the demands on teachers, schools and students, flexible, tailored professional development such as the materials developed can help to address the difficulties encountered by many teachers and schools in accessing high-quality mathematics-specific professional development.

Many of the features of language-responsive mathematics teaching are easily embedded within existing resources that support mathematics teaching and learning. As departments continue to develop the resources they use and the classroom practices they are focusing on, the guidance offered as part of the professional development materials can support the wider implementation of language-responsive design principles.

The materials developed as part of this project are freely available at:

https://www.education.ox.ac.uk/project/developing-language-responsive-mathematics-classrooms/

1. Introduction

The project set out to develop a professional development programme for secondary mathematics teachers focused on addressing the linguistic challenges frequently encountered in the learning of mathematics. The project also aimed to research the impact of this programme on the classroom practice of the teachers involved in the development of the programme and their students' learning. The project focused on the teaching of three Key Stage 3 topics: linear equations, angle properties of parallel lines, and probability.

Language-responsive mathematics teaching is defined as teaching that integrates and enhances mathematics and language together, amplifying rather than simplifying language (Erath et al., 2021; Prediger & Neugebauer, 2021b).

More generally language-responsive teaching includes using literacy strategies, focusing on mathematical communication, as well as making use of informal languages and home languages. Language has both a communicative and an epistemic function in mathematics classrooms (Moschkovich, 2015; Pimm, 1987) and these functions can be intertwined, reinforcing or in tension with each other.

The majority of the existing research on language-responsive mathematics teaching focuses on addressing educational disadvantages faced by multilingual learners who are learning the language of instruction at the same time as learning mathematics (e.g., Lenz et al., 2024; Prediger & Neugebauer, 2021a). However, there has also been much promising research suggesting that the approach can be beneficial to all students who face language difficulties when learning mathematics (Barwell et al., 2016; Prediger et al., 2022; Prediger & Neugebauer, 2021a).

The focus of the professional development programme involved four key aspects:

- identifying topic-specific meaning-related language that emphasises the underlying mathematical concepts and relationships
- adapting and using mathematically demanding tasks that offer students opportunities to reason and explain their reasoning
- developing norms and routines that support students' mathematical communication
- using and connecting multiple representations that illustrate the underlying mathematical structure, relationships or concepts.

These key aspects originated in the review of research conducted by Erath et al. (2021) which identified six design principles established in research as enhancing language for mathematics learning. These design principles evolved during the workshops with the mathematics teachers primarily to reflect the specific context and focus of the professional development programme.

Language-responsive mathematics teaching is teaching that integrates and enhances mathematics and language together, amplifying rather than simplifying language

Role of language-responsive mathematics teaching in addressing educational disadvantages

There are several ways in which the linguistic challenges students encounter when learning mathematics explain or amplify wider systemic educational disadvantages. Many of these relate to how academic language may reproduce inequality through implicit expectations of academic language use necessary for learning and assessment, and assumptions that this will develop without explicit attention, i.e., it is part of the hidden curriculum in schools (Heller & Morek, 2015).

The difference in attainment between socioeconomically disadvantaged students and their peers widens over schooling (Kaye, 2023), and by age 15, the difference in mathematics attainment is around the equivalent of 1 year of additional schooling between students entitled to free school meals and those who are not in England (Ingram et al., 2023). For students who speak a language different from English at home, this difference is around the equivalent of 3-4 months of additional schooling. These students can also benefit less from mathematics teaching than other students (Lenz et al., 2024).

All students need language skills to learn mathematics and to participate in the mathematics classroom. Most research on language-responsive mathematics teaching has focused on forms of linguistic disadvantage that students have, which include multilingual background, low proficiency in academic language in the language of instruction and lower prior academic attainment (Prediger, 2022).

In this project, we shifted the focus to the linguistic challenges students might encounter when learning mathematics, such as the vocabulary and grammatical structures used, the conceptual density of the key ideas being communicated, and the opportunities available to develop and refine mathematical communication (Schleppegrell, 2007). This shift is important as it places emphasis on what teachers can do to address the linguistic challenges that arise within mathematics or in the teaching of mathematics rather than on a deficit of the students.

Learning mathematics involves communicating ideas, sharing strategies and reporting solutions. Students need to engage with others through speaking, listening and communicating about mathematics. Learning mathematics also involves thinking with and through language. Language-responsive mathematics teaching focuses on developing students' ability to use language to think and communicate about mathematics. Drawing on prior research that identified design principles for language-responsive teaching (Erath et al., 2021), the project developed a model of four dimensions of planning and teaching for language-responsive mathematics teaching (Figure 1):

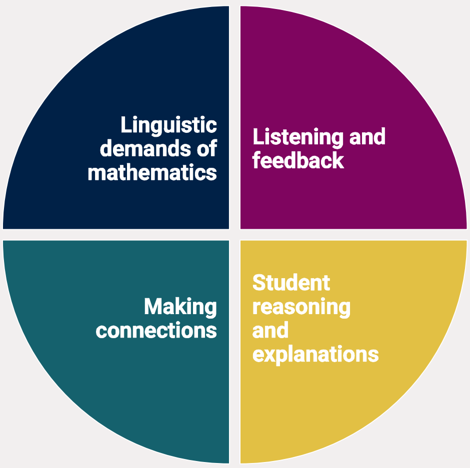


Figure 1: The four dimensions of language-responsive mathematics teaching

2. Methods

The project was a collaboration between the project team and mathematics teachers teaching Key Stage 3 in state schools. It focused on three key mathematical topics. The aim of the project was to design professional development materials to support language-responsive mathematics teaching.

Research aims and questions

The initial questions of the study were:

RQ1: How can mathematics teachers support students facing linguistic challenges when thinking about and working on mathematically demanding tasks?

RQ2: How can mathematics teachers support students to engage in mathematical classroom interactions around these demanding tasks?

As we began to work with the teachers and analyse the data, these questions were developed to be specific to the different topics, design principles and teacher moves. For example, RQ1 was considered in terms of how mathematics teachers can support students facing linguistic challenges in the topic of linear equations when working on mathematically demanding tasks that encourage them to participate in explaining, arguing and reasoning.

Areas of mathematics studied

The project focused on three areas of the Key Sage 3 (lower secondary) mathematics curriculum in England: linear equations (algebra), angle properties (geometry), and probability (statistics). These were chosen because they involve underlying concepts that are fundamental to understanding mathematics (Watson et al., 2013) and involve contrasting styles of argument structure and use of representations.

The project team and participating teachers co-designed guidance materials for demanding and rich classroom tasks to support teachers and students in overcoming linguistic challenges associated with learning in these areas of mathematics.

There is a wealth of research into the teaching and learning of these, but there are also distinctions between them in argumentation structures, uses of representations, and connections to everyday experiences.

Teacher collaborators, schools and recruitment

Initially, 11 mathematics teachers from 6 state schools in England were recruited to participate in the project programme of PD and co-develop the lesson materials. These schools included two coastal schools, one of which is a grammar school, in the East and Southeast, and four inner-city comprehensive schools in different cities in the Midlands. However, shortly after the start of the project, one school withdrew following a change in Headteacher and another withdrew following a poor Ofsted outcome. Two mathematics teachers from one further inner-city comprehensive school in the West Midlands were subsequently recruited, joining the project after the first design cycle.

Mindful that teachers may have changed roles between academic years, at the start of the academic year 2023-2024, we reconfirmed participation with teachers and schools. All teachers continued to participate; one teacher had moved schools, but their new school, another comprehensive school in the same area, agreed to participate in the later stages of the project.

Recruitment of teachers was through

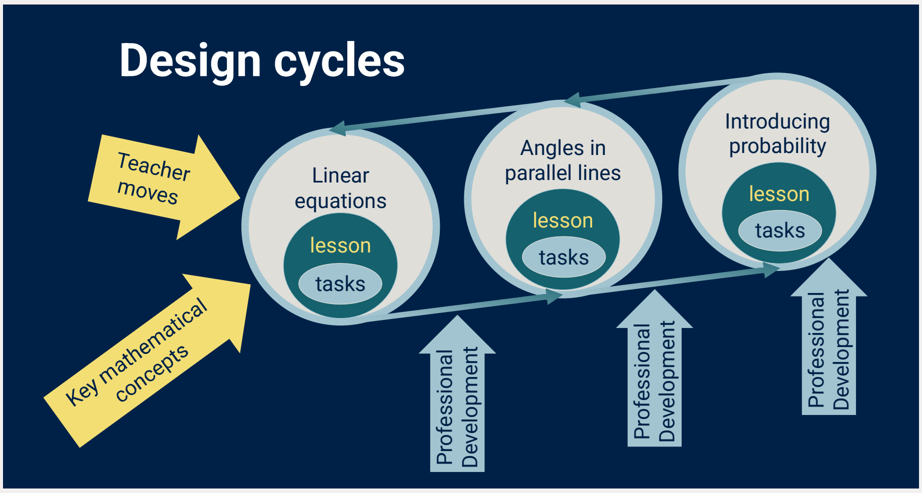


Figure 2: The design cycles

advertising for participants on the project's website and through teacher subject associations (MA, ATM), NCETM and professional contacts of the project team. One teacher was recruited at a research conference, though they did not have a research background in language. Participating teachers were self-selecting. Some of the teachers volunteered because of an interest in the language aspects of mathematics teaching, while others were interested in engaging in research related to classroom practice more generally and did not have a specific prior interest in language.

The collaboration process also drew from four of the PD design criteria developed by Clark-Wilson et al. (2015) and was achieved through a pedagogy based on the Discipline of Noticing (Mason, 2002) using the design principles for language-responsive mathematics teaching (Erath et al., 2021) (see below for more details).

These design criteria draw extensively on the expertise of teachers, using school-developed assessments and resources, assimilation with the school scheme of learning, and embedding peer support. These aspects of PD are essential for teachers to be able to connect to their local context. The project team worked with two teachers from each school, except in one case where this was not possible.

Throughout the development of the professional development, the mathematical concepts and processes had primacy in the choice and design of the materials and accompanying pedagogy. The language-responsive design principles were used to then highlight the role of language in learning these concepts and processes.

Work with teachers on the co-design of materials

The project team worked with the teachers between February 2023 and July 2024. This work included ten online twilight sessions focused on mathematical tasks and four whole-day hybrid sessions held at the University of Oxford. The twilight sessions focused on the linguistic demands of familiar classroom tasks and the activities around them (described below). The whole-day sessions focused on pedagogy related to teacher moves, using the lesson videos collected as part of the project.

The project used a design-based research approach consisting of cycles of design, teaching and analysis widely used within mathematics education research (Cobb et al., 2003), illustrated in Figure 2. The design phase of each cycle drew on existing relevant research, teacher education materials, and participating teachers' experiences and expertise to design a sequence of activities within a lesson focused on a key concept within the topic (see Section 3). The teachers then used the designed activities and materials in their own classrooms, and video-recorded one of these lessons. Subsequent cycles of the design process involved revisiting the materials developed by jointly watching extracts from the video recordings and revisiting the tasks during the PD sessions. Figure 3 illustrates the contents of the linear equations design cycles.

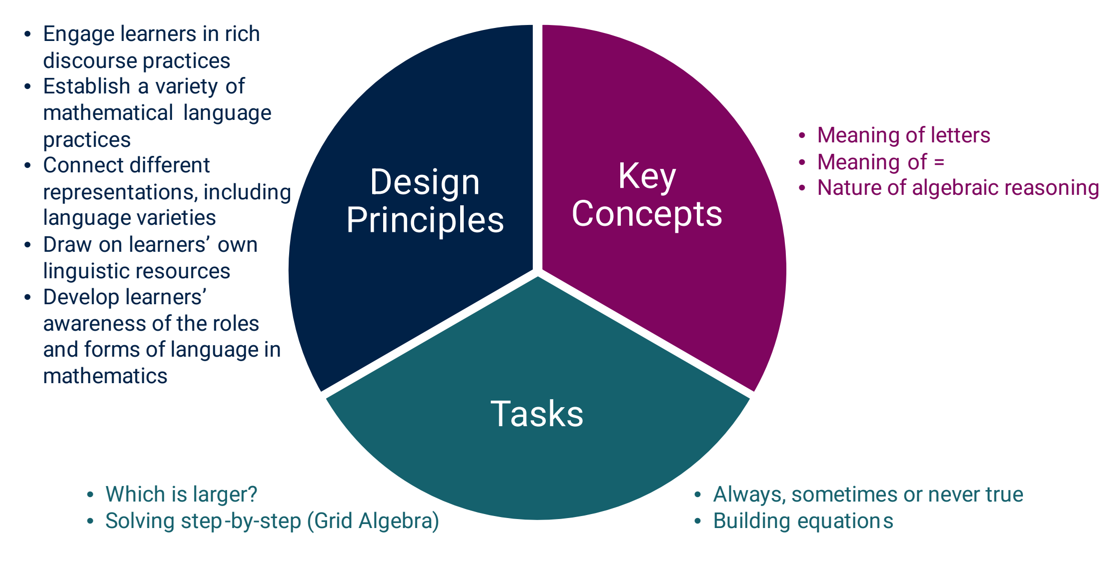


Figure 3: Contents of the linear equations design cycle

Materials development: how the Design principles evolved

Erath et al. (2021) identified six design principles (DP) for mathematics teaching that enhances language mathematics learning. These principles were based on a review of recent research and included:

1. Engage students in rich discourse practices
2. Establish various mathematics language routines
3. Connect language varieties and multimodal representations
4. Include students' multilingual resources
5. Use macro-scaffolding to sequence and combine language and mathematics learning opportunities, and,
6. Compare language pieces (form, function, etc. to raise students' language awareness (p. 247).

During the project, these six principles were developed in three ways. Firstly, to address the specific context of secondary mathematics classrooms in the England context. Secondly, to make them more accessible to mathematics teachers working in this context. Finally, to reflect the project's broader focus on all students that face linguistic challenges when learning mathematics, including students with a history of low attainment, those less familiar with the academic language of the classroom as well as multilingual students.

This development resulted in four dimensions of planning and teaching for language-responsive mathematics teaching (see Figure 1):

- Linguistic demands of the mathematics
- Listening and feedback
- Student reasoning and explanations
- Making connections

Linguistic demands of the mathematics

This dimension focuses on DPs 5 and 6, but also includes aspects of DP3 – through connecting visual and verbal representations. Linguistic features such as lengths of words and sentences, passive constructions, comparative structures or conditional clauses – can mean that the mathematics in focus is both mathematically complex (cognitively demanding) and linguistically complex (Abedi & Herman, 2010; O'Halloran, 2005). One aim of this project was to explicitly address this linguistic complexity in ways that maintain the mathematical core ideas and processes.

The first aspect of addressing this linguistic complexity was to identify meaning-related vocabulary and phrases. These are shared words and phrases that students use to communicate and discuss conceptual meanings before developing more technical and formal mathematical ways of communicating (Pöhler & Prediger, 2015; Prediger & Neugebauer, 2021a).

A second aspect focused on the different forms of mathematical communication. In our communication about this dimension with teachers, we said:

Communication in mathematics lessons consists of images, diagrams, notations and other ways of communicating about mathematical objects, ideas, and practices. Naming or labelling features of a diagram can be done in a variety of ways including using words, labels, notations, or colours. When names are attached to something mathematical, whether that is an object such as a number or angle, a relationship such as parallel or equal, or a process such as conjecturing or generalising, these names can be used to communicate about that object, relationship or process. Naming can also emphasise the importance of a particular action or object; for example, naming auxiliary lines highlights them as something mathematicians may use in a geometric image to work with that image.

The choice of the words used in teaching can also influence the meaning that is being emphasised. For example, 2/3 can be described as two thirds, two out of three, two divided by three, two over three. Each of these conveys a different meaning, which may be more useful in some contexts than others. Two thirds emphasises the fractional structure, two of these things called 'thirds', as well as the number it represents. Two out of three may be more useful in probability contexts where it emphasises the number of successful outcomes out of the total number of outcomes. Two divided by three emphasises the process involved, for example when converting to a decimal or when sharing or grouping. Two over three emphasises how to write this fraction.

Keeping this dimension in focus may lead to noticing linguistic demands of what the teacher says or writes, or what the students say or write, or a combination of these.

Listening and feedback

This dimension focuses on DP1 but with a focus on listening to and responding to these student discourse practices such as explaining meanings, argumentation and justifying conclusions. In responding to students this dimension also incorporates DPs 3 and 5 and 6. In particular, DP5 involves planning for and anticipating student responses, and DP2 includes establishing a routine of students listening to each other and building on each other's ideas.

In our communication about this dimension with teachers, we said:

Listening can take different forms and can have different roles in mathematics lessons. Teachers listen to their students, students listen to their teacher, and students listen to each other. There is a difference between listening for something in particular (a specific answer to a question) and listening to something. Another distinction can also be made between listening to something in order to evaluate or interpret it (i.e. to understand what students are thinking) and listening to something in order to make connections to other ideas and descriptions and also to develop a teacher's own understanding of the mathematical learning.

It can be very hard to 'see' listening. Listening for something can be seen when mathematically appropriate responses are given by students but are not used because they do not fit with the particular focus of the task, discussion or lesson. Teachers often notice they are listening for something when they get an answer and are confronted with a thought such as "that was what I was looking for", or "that's not what I was expecting" – it is the sense of expectation that is often associated with listening for something. Listening to something can sometimes be seen through what follows. Are the same words used, are questions asked about what was said whether these questions are for clarification or to develop an idea etc further.

Student reasoning and explanations

This dimension includes DPs 1 and 2 through the rich discourse practices of reasoning, explanations and argumentation and development of these through establishing mathematics language routines. It also includes DP3 in that informal and formal language are both considered – the dimension involves identifying reasoning and argumentation structures irrespective of whether the language variety used is informal or formal, and DP4 as students may utilise multilingual resources in their practices of reasoning and explaining.

In our communication about this dimension with teachers, we said:

Reasoning, explanations and argumentation demand particular structures in mathematics that makes them distinct from explanations and arguments in other subjects. They may involve using definitions, being precise, working deductively, identifying and describing patterns and structures, as well as drawing a conclusion. Even explanations or arguments that draw a false conclusion contain mathematical features such as making a conjecture, making a connection between the problem and a previous problem, using mathematical notation or terms, comparing different solutions or solution strategies.

Noticing these mathematical features can help students make sense of what a mathematical argument or explanation is, and to identify how they can improve their own explanations or arguments.

It can take time to construct and articulate an explanation or argument.

Making connections

This dimension draws on DP3 – connecting language to the mathematical concepts, connecting more informal language varieties to more technical ones, and making connections between representations. It may, therefore, also similarly draw on DP4 by connecting different languages. The dimension also draws on DP6, as a means of scaffolding connections between language varieties and raising language awareness of similarities and differences through comparing alternative phrasings.

In our communication about this dimension with teachers, we said:

Making connections is a key part of teaching and learning mathematics, but there are many different types of connection that could be made (and not all of them are mathematical). Connections that relate to communicating mathematics include between representations, between families of words, between informal ways of describing mathematics and more technical ways, as well as connections between ideas, tasks, methods, or processes. Making connections involves more than experiencing two solution methods, two different representations, two examples alongside each other as it needs the similarities between these to be noticed.

These connections can be made by the teacher and/or the students, and they can also be articulated by the teacher and/or the students.

Task selection

Teachers provided tasks from their schemes of learning, which indicated some commonalities across schools. Written permission was sought from the original authors or designers of the tasks provided, where this was not achieved (largely from no reply to email), similar tasks from authors and publishers where permission was granted were substituted.

The tasks provided by teachers included reasoning tasks and exercises designed to develop procedural knowledge. Tasks for development by the project were chosen within each topic to ensure that there was variety in the student activities required, such as giving reasons or explanations, representing mathematics fluently and formally, applying procedures and processes, or arguing and justifying.

Tasks were worked on collaboratively during the twilight PD sessions, identifying the language-responsive aspects of these tasks drawing on the four dimensions. The teachers then used these tasks in their teaching and the videoed lessons focused on the use of one (or sometimes more) of these tasks during a lesson, but the variety of project tasks was not necessarily captured in the videos.

Data collection

All meetings and professional development sessions were recorded. Online and hybrid meetings were video recorded using Microsoft Teams and in-person meetings were audio recorded.

Data collection: video recording

All video-recorded lessons included at least one of the tasks considered in the twilight PD sessions. Teachers nominated these lessons for video recording based on when the topics fitted within their school’s scheme of learning. Whole lessons were recorded not only to capture other types of tasks and activities for comparison, but also because the tasks chosen to be used were not necessarily representative of the types of tasks used in the twilight PD sessions.

Data collection: student assessments

The project team developed assessments for each topic based on the assessments, worksheets and textbooks used in the schools. These were to be taken within mathematics lessons by an intervention class (i.e. a class whose teacher was participating in the project PD) and a comparison class in the same school year group and of similar prior attainment in the school, after both classes had studied the topic. These assessments were only used as post-tests. The research design choice not to use pre-tests was taken to minimise the burden of testing, while still providing teachers with useful information about their students' scores, and to provide comparison data between classes within the same school.

The majority of teachers provided the project team with copies of their usual assessments. Assessment items were developed by the project team based on these items and similar items from the worksheets and textbooks used in the schools, as well as standard KS3 tests on each topic, likely to be familiar to teachers. It was not always possible to directly use items from the teachers' own schools due to difficulties securing permission for use where these were published assessments.

As the intervention classes were in Years 7, 8, or 9, the assessment items spanned a range of difficulty but were intended to be accessible to students in KS3. To reflect the aims of the project, items were developed to offer opportunities for student reasoning, and several items taken from the school assessments and KS3 items were adapted to include requests for students to explain their answers. They were not, however, designed to be close to the project tasks used in lessons with the intervention class.

Schools had choices over when to use the assessments and whether these would be marked by the teachers or members of the project team. The majority of assessments were marked by the project team.

Not all participating schools were able to complete all the assessments, and not all teachers completed the assessment with a comparison class. Overall, we have anonymised assessment scripts from teachers at four of the schools for at least one of the topic areas. Factors that affected the availability of assessment data for the project included: probability being the last topic covered by the project, so there was limited time in the school year, assessments needing to be cancelled due to unanticipated school events, and teachers moving schools so it was no longer possible to share the assessment data with the project team.

Data analysis

Data analysis: video analysis

All the video recordings of the teachers implementing the tasks and accompanying activities and teacher moves were analysed in three stages.

Stage 1 focused on dividing the videos into task-focused episodes. These task-focused episodes included the introduction of the task, students working on these tasks, any whole-class discussions or small group discussions, and the conclusion of the task.

In Stage 2, each episode was coded by two members of the project team according to four of the domains of the Global Teaching Insights observation framework (Bell et al., 2020). These four domains focus on the mathematical content of the episode (Quality of Subject Matter and Students' Cognitive Engagement) and the nature of discourse (Assessment of and Responses to Student Thinking and Classroom Discourse). The observation scales are available from the Global Teaching InSights website: https://www.oecd.org/en/about/projects/global-teaching-insights.html

Only the higher-inference coding framework from the Global Teaching Insights materials was used. Each domain was divided into three components, rated on a scale of 1 to 4, where 1 referred to the low quality or quantity of the target component, and 4 was the highest quality or quantity. In the original Global Teaching InSights study, episodes were identified by length rather than task focus, and this length was longer than all the episodes analysed in this study. Consequently, the episodes in the Global Teaching Insights Study are likely to include a wider variety of teaching practices and, therefore, are likely to score more highly in components that measure the frequency of an event.

One member of the project team was a *master rater* for this observation framework as a result of leading the original study in England. Two other members of the team were then trained to use this observation framework, using the published training materials supplemented with video extracts from this study.

All videos were rated by two trained raters. Overall, there was 89.4% exact agreement between the two raters and 98.5% exact or adjacent agreement between the two raters. Where there was more than 1 point difference between raters, the episode was jointly discussed until agreement was reached.

Only episodes that included some interactions around the mathematics in the lesson were analysed. Episodes where lessons were interrupted or where the register was taken (unless the students were working on individual maths tasks at the same time), for example, were not included in Stage 2 of the video coding.

Stage 3 coding focused on teacher and student moves in mathematical classroom interactions around demanding tasks. The intention in performing this coding was to describe the different frequencies of the targeted teacher moves, to capture some measure of student reasoning and participation, and to explore the influence of teacher moves by looking for patterns between teacher and student moves.

Stage 3 coding was only carried out on sections of whole-class discussion within the twenty episodes which had scored highly on explicit student thinking during Stage 2 coding. The main reason for this focus was that the teacher and student moves central to Stage 3 coding are not of great relevance during individual seat work or teacher presentations, but are meaningful in application for periods of whole-class interaction.

The coding process started with six codes for teacher moves taken from Erath et al. (2021), and six codes for student moves drawn from Drageset (2015), Erath et al. (2018) and Erath (2017). A small amount of initial coding was done using part of one episode at a face-to-face meeting with a small group from the project team, which led to an additional preliminary code for a teacher move. Two members of the project team then coded three entire episodes independently before comparing their responses, co-constructing a codebook, confirming the use of this additional code for a teacher move and supporting two further additional codes for student moves. These two researchers then continued to code half of the data each, with episodes allocated to ensure a spread of teachers and schools as far as was possible, and the coders jointly resolving any challenging or ambiguous cases.

Further details of the codes and the results of Stage 3 analysis are presented below.

Data analysis: student assessments

For the student assessments, comparisons were made between the intervention classes and the comparison classes within the same school, but not across the different schools. This was for two reasons. Firstly, the analysis was unable to take into account the prior attainment of the classes and consequently classes in different schools were not comparable, whereas the comparison classes within schools were designed to be comparable. Secondly, the student assessments were designed to fit within the normal assessment practices within each school, and therefore minimise the testing burden for both teachers and students. This resulted in some questions being omitted from some tests where they did not fit with the school's scheme of learning.

For each student, the total number of marks in the assessment, the number of reasoning questions answered and the total marks in the reasoning questions were calculated. T-tests were performed where the total marks or the total reasoning marks suggested there may be a significant difference between the comparable classes. Item level analysis were also performed for the reasoning questions. In this report only the overall results of the quantitative analysis are reported.

Research Ethics

The research project was granted approval by the Department of Education Research Ethics Committee at the University of Oxford. Research followed BERA guidelines (2018), which were valid at the time of ethical approval.

The project required Headteacher consent and teacher consent. Student consent was based on an opt-out arrangement except where a school requested opt-in consent. Consent, data storage and analysis followed the protocol as agreed with the Department of Education Research Ethics Committee at the University of Oxford. All teachers, teacher educators and students' names in this report are pseudonyms. In order to preserve anonymity some findings are reported in more general ways than the data collected so that teachers and schools are not identifiable from the data reported.

Throughout the project, members of the project team were mindful of potential harms arising from teachers participating in the project. For example, we made arrangements for PD sessions in consultation with the participating teachers to fit with their schedules and with awareness of pressures during school terms. Also, we recognised that asking experienced classroom teachers to reflect on their practice and work together on demanding mathematics tasks might touch on issues relating to their expertise and identity as mathematics teachers. This was particularly relevant when discussing extracts from the video recordings during the meetings. These extracts were chosen to stimulate discussion around students' reasoning or difficulties in relation to a task, rather than to exemplify particular teaching practices.

The project was also underpinned by the principles within the Discipline of Noticing (Mason, 2002) which focus on developing awareness and self-reflection, and using this awareness to make conscious decisions relating to practice. For example, in keeping with these principles, PD sessions and task guidance drew attention to opportunities and challenges afforded by the tasks when used in certain ways, but did not prescribe particular ways of acting beyond the general principles for language-responsive mathematics teaching.

3. Material development

In collaboration with teachers, the project team developed research-informed materials to support language-responsive mathematics teaching. These included teacher guides for familiar tasks and more theoretically-oriented guides to language-responsive teaching and research on concepts such as *fairness* and *likelihood* for probability, the *nature of angles* for angles in parallel lines and *meanings for the equals sign* and *variables* for algebra. The project team also developed a framework to support observation and discussion about language-responsive teaching.

The materials (for examples see Figure 4, Figure 5, Figure 6 and Figure 7[1]) were developed iteratively, informed by design principles and teacher moves for language-responsive teaching (Erath et al., 2021), videos of classroom use, and feedback on format and content from teachers at PD sessions, as outlined in the Methods section.

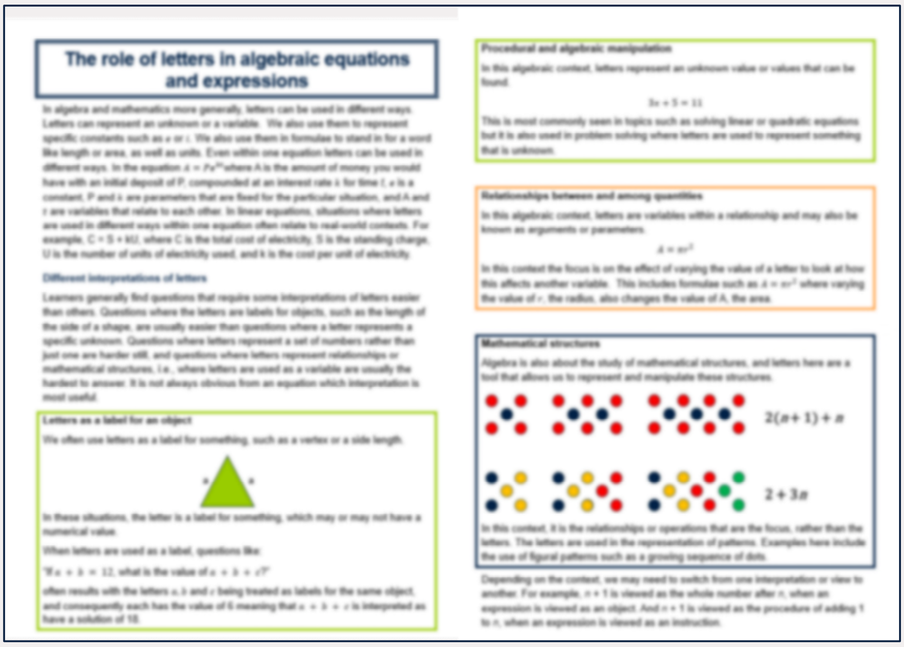
One core principle when developing the materials was not to tell teachers what they should do. Instead, the aim was to support teachers to reflect on their practice and develop awareness of the role of classroom language in student learning, for example, when responding to student reasoning. Another principle was taking a non-deficit stance when sharing the research on topic-specific challenges. These were framed as situated within the mathematics, rather than as deficiencies of students. That is, emphasising why mathematics is hard to do, rather than what students can't do.

The theoretical guides (see example in Figure 5) are illustrated by examples pertinent to the lower secondary level and the mathematical topics. They include research references, primarily to open access references. Our intention is that these guides could support departmental discussion of how classroom language use is involved in teaching these concepts as well as supporting individual teachers to notice subtle aspects of their own and students' language use and to use this developing awareness in their teaching.

$$x = 3$$
$$x + 2 = ?$$
$$x + 2 = 3$$
$$x + 2 = 2 + x$$
$$x + 2 = x + 3$$
$$x^2 = 3$$
$$x + y = xy$$

Figure 4: Extract from the guide to the different meanings of '='

[1] The text in Figure 5, Figure 6 and Figure 7 has been obscured due to copyright restrictions.



The role of letters in algebraic equations and expressions

Different interpretations of letters

Letters as a label for an object

Procedural and algebraic manipulation

Relationships between and among quantities

Mathematical structures

Figure 5: Example pages from the guide for language-responsive teaching

Task guides are also based on underlying research about language-responsive teaching and concepts that relate to the task and topic. Task guides included the published tasks (reproduced verbatim for copyright reasons), and sections of the task are interspersed with task-specific guidance. Through this additional text, the task guides aim to relate specific details of opportunities and challenges of the task to the four dimensions of language-responsive teaching (indicated by colour coding, see Figure 6). For example, one task guide provides different ways students and teachers might say algebraic expressions such as $x + y$ and xy (Linguistic demands of the mathematics), it draws attention to important distinctions between reading out statements and what they mean (Listening and feedback) and how students might reason about variables in algebraic expressions (Student reasoning, explanations and argumentation). Mindful of teachers' time, in addition to the detailed support for classroom use, the task guides include a two-page 'quick guide' as a reminder of core ideas for the task related to each dimension.

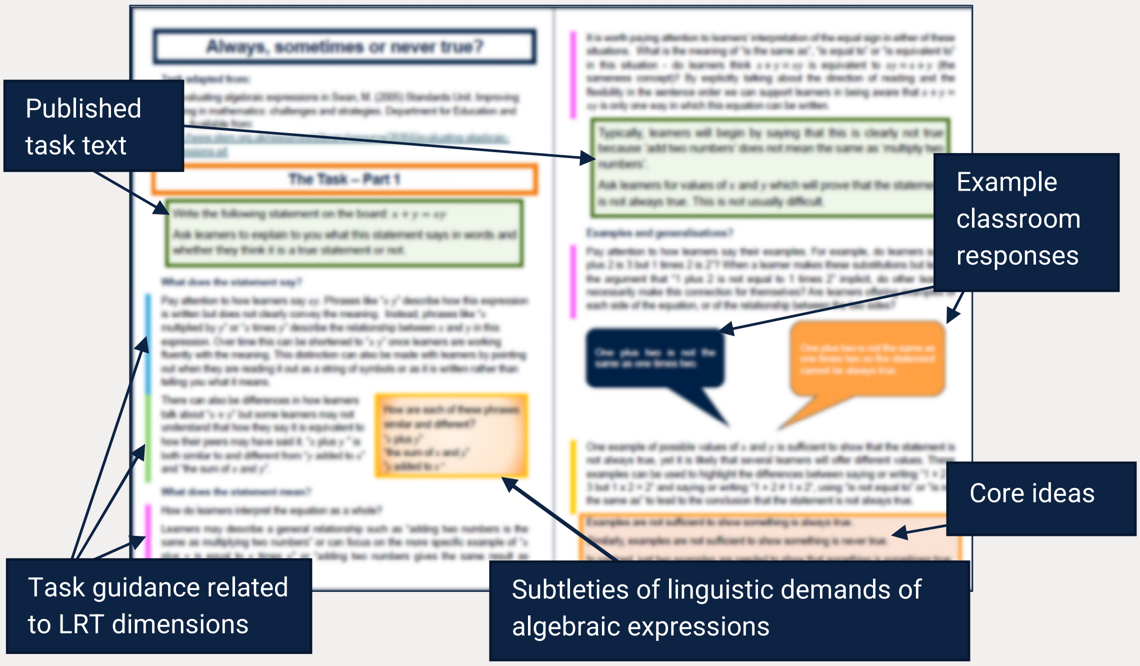


Figure 6: Structure of task guidance

As noted above, the project team also developed a framework to support discussions about language-responsive mathematics teaching through peer observations and conversations, focusing on what is noticed in the lessons in relation to the four dimensions of language-responsive teaching. Within each dimension of the framework (each column in Figure 7) there are several examples of teacher and student moves that relate to that dimension. For example:

- The teacher and/or students have the opportunity to speak or write using mathematical language themselves (Linguistic demands of the mathematics).
- Feedback on explanations and justifications focuses on the quality of the mathematics (Listening and feedback).
- The teacher or students draw attention to the mathematical features of an explanation, justification, or argument (Student reasoning, explanations and argumentation).
- The teacher and/or students make connections between different representations (Making connections).

The framework also lists over 40 student activities that might be noticed in the lesson, such as clarifying, deciding, exemplifying, generalising, justifying, listening, naming, reasoning, refining, varying, verifying and writing. These were largely taken from work on the nature of mathematical thinking (e.g., Mason, 2006; Mason, 2011; Mason et al., 2010). These activities were included to highlight the wide range of types of mathematical activity students could be engaging with. They were used to identify the combinations of activities that might or did result from students working on the tasks being developed.

In keeping with the principles of supporting noticing and taking a non-deficit view, the framework emphasises that conversations should focus on what was noticed during a lesson or activity, rather than what was missing or might have happened. This is intended to help develop understanding of the nature of language in our teaching and its role in students' learning while also avoiding judgments about the teaching observed.

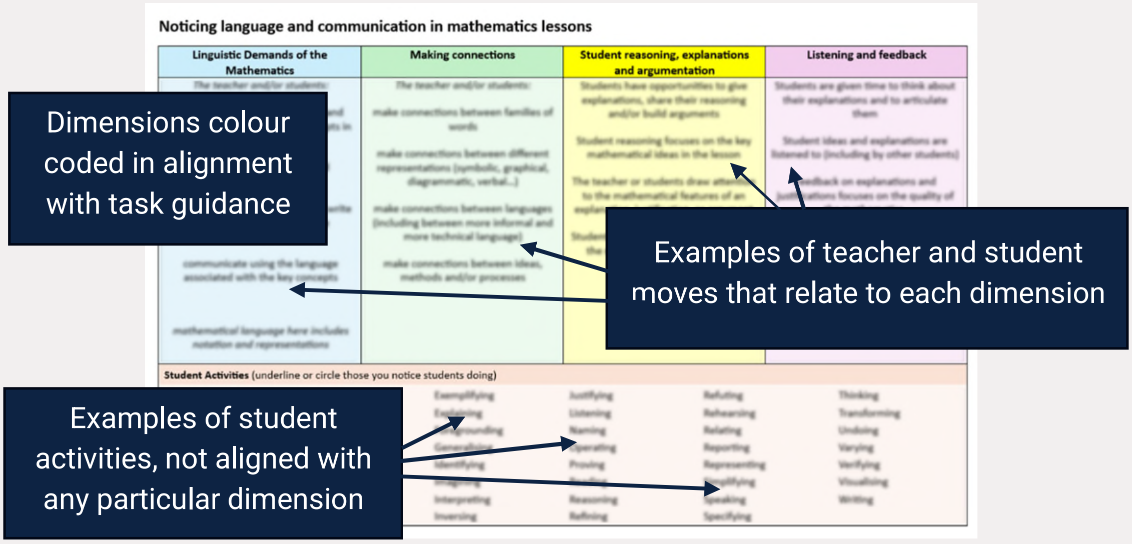


Figure 7: Framework for noticing language and communication in mathematics lessons

For ease of sharing, accessibility, and managing feedback on version updates, materials were shared with the participating teachers as documents (pdf). Currently the materials need to be closed but they are also available in online versions that will maintain availability and make use of interactive features of the online platform. This online version can continue to be developed beyond the scope of this project.

4. Teaching practices

A key part of the project was to develop particular teaching practices established in research to support students' learning in general and in mathematics specifically. Videos from one lesson within the unit that included the focal topic were analysed in three stages, as described in the methods section.

A total of 49 episodes were identified with between 3 and 10 episodes in a lesson. The episodes ranged in length from half a minute to slightly over 29 minutes. Twenty of these episodes included tasks from the project with 27 including other mathematical tasks. Two episodes did not include the students doing any mathematical activities as they involved packing away at the end of the lesson, so they were not included in the Stage 2 or Stage 3 coding.

Components of teaching

Table 1 shows the overall results of Stage 2 of the video coding. The average ratings for episodes that included tasks discussed and worked on with the teachers in the PD sessions were higher in every component than the average ratings for the other episodes in the lessons. They were also higher for every component than the average ratings reported by the Global Teaching InSights project for teachers in England (Ingram et al., 2020). In particular, the average ratings for the Nature of Discourse, Engagement in Cognitively Demanding Subject Matter, Multiple Approaches to and Perspectives on Reasoning, Eliciting Student Thinking, and Teacher Feedback were all over 1 scale point higher in the episodes including PD tasks than those that did not include these tasks.

Table 1: Average ratings for Stage 2 coding

Component	PD tasks (Mean, SD)	Other tasks (Mean, SD)
Nature of discourse	3.11 (0.89)	1.96 (0.51)
Questioning	3.26 (0.73)	2.33 (0.80)
Explanations	3.08 (0.79)	2.31 (0.79)
Explicit connections	2.32 (0.87)	2.15 (0.88)
Engagement in cognitively demanding subject matter	3.45 (0.81)	2.00 (0.82)
Multiple approaches to and perspectives on reasoning	2.66 (1.19)	1.44 (0.92)
Understanding of subject matter procedures and processes	3.03 (1.09)	1.81 (0.87)
Eliciting student thinking	3.32 (0.80)	2.27 (0.68)
Teacher feedback	2.66 (1.17)	1.75 (0.72)
Aligning instruction to present student understanding	3.67 (0.69)	3.02 (0.91)

Note: ratings are on a scale of 1 (low quality or quantity) to 4 (high quality or quantity)

The majority of the tasks that the teachers chose to use during the lessons that were video recorded focused on eliciting students' reasoning and were designed to provide opportunities for students to explain why, rather than explaining how or that. This partially explains the higher average ratings for the Nature of Discourse, Eliciting Student Thinking and Engagement in Cognitively Demanding Subject Matter which measure different aspects of students' reasoning. In particular, Eliciting Student Thinking records the level of detail in students' responses to questions and tasks.

Tasks that prompt students to share their reasoning are likely to prompt more detailed responses than tasks that require the reporting of an answer. The reporting of methods or approaches, however, would also rate highly in this component as these also require detailed responses from students, as the nature of these responses is considered in the Engagement in cognitively Demanding Subject Matter component.

Although the PD did include some tasks that focused on practising procedures and methods, these tasks were not used by the teachers in the recorded lessons. However, many of the episodes that did not include the PD tasks did focus on students practising these procedures or methods. This may partially explain the lower average rating in these episodes for the Nature of Discourse, Questioning and Understanding of Subject Matter in particular. These episodes had lower average ratings in these components than those reported in the Global Teaching Insights study.

There were examples of the specific teacher moves considered in the PD in the episodes that did not focus on the tasks considered in the PD, suggesting that there is some transference of these moves into the teachers' wider practice. This aspect is analysed further through Stage 3 of the video coding.

Teacher and student moves

The Stage 3 coding process started with six codes for teacher moves (TM1-TM6), taken from Erath et al. (2021) shown in Table 2, and six codes for student moves (SM1-SM6), drawn from Drageset (2015, 2021), Erath et al. (2018) and Erath (2017) shown in Table 3. Initial coding led to an additional preliminary code for a teacher move, TM7. Further coding confirmed this use and supported two further additional codes for student moves, SM7 and SM8.

Table 2: Codes for teacher moves

Code	Description
TM1: Plan and prepare collective discussions that focus on mathematical concepts	This was often coded once at the start of an episode for recognizing that the teacher used a task suitable for stimulating mathematical discussion, but was also used when a teacher moved around the classroom, actively gathering student ideas in a way that would inform future collective discussion.
TM2: Understand and connect students' ideas and mathematics; make it accessible to as many students as possible	Here the teacher might be exploring the students' thinking, using a guiding question, or bringing together and supporting students' mathematical ideas by revoicing or using language supports.

TM3: Enhance language practices for learning mathematics	This differs from TM2 in that the teacher is developing or bringing attention to the language register or repertoire in some way.
TM4: Encourage participation in demanding discourse on mathematics	This was coded when the teacher explicitly asked the students to move beyond narration or description; instead, the students were invited to explain, argue, or give a reason.
TM5: Pay attention to your feedback and evaluation of students' mathematics	This code indicates when a teacher moved beyond simple evaluation of a student response, perhaps by moving into joint examination of a student statement, or initiating self-repair.
TM6: Purposefully use pauses and silence	TM6 indicates when a teacher plainly used a pause to give students time to articulate their thinking.
TM7: Invite students to read something out loud	Although this new code can be related to TM2/3, it indicates when a teacher specifically asks a student to describe something visible (and typically shared) in their own words.

A more detailed description of the codes and the supplementary guidance used by the project team, along with examples and non-examples, is provided in Appendix A.

An example from the results of the coding is shown in Table 4 and the full results are provided in Appendix B.

Table 3: Codes developed for student moves

Code	Description
SM1: Short answer	This code includes single word responses such as 'yes' and 'no'.
SM2: Question	The student asks a question to do with the mathematics.
SM3: Explain processes	The student explains how they got to an answer, perhaps laying out a step-by-step calculation. This can be thought of as explaining *how*.
SM4: Explain concepts	The student explains a mathematical concept. This can be thought of as explaining *what*.
SM5: Explain reasons	The student shares the reasoning behind their answer. This might build on a calculation (SM3) but would typically involve connecting vocabulary such as "because" or "but". This can be thought of as explaining *why*.
SM6: Describe	The student reads or describes some mathematics which is typically visually present in the classroom.
SM7: Engage in a teacher-style move	The student uses a move usually enacted by a teacher.
SM8: Choral response	The students offer individual responses at the same time, for instance by volunteering answers on mini whiteboards.

There were a high number of student moves, with almost as many student moves observed (424) as teacher moves (489) in total, which speaks to student engagement during activities of this type. The second most frequently coded type of student move was the SM5, wherein students shared some reasoning, offering often linguistically demanding comments which spoke to their mathematical thinking, such as *"Because you can't really change the value of q… because if q equals one, then you can't change it halfway to suit the question."*

Table 4: Example from results of coding teacher and student moves

Sch	Teach-er	Epi.	TM1	TM2	TM3	TM4	TM5	TM6	TM7	SM1	SM2	SM3	SM4	SM5	SM6	SM7	SM8
S1	Ben	E1	4	13	6				1	15	4		1	6	5	2	2
S1	Alex	E1	1	2	8	1		2	7	3	1			4	10		
S1	Alex	E2	2	22	7	4				17	1	4		13			
S1	Alex	E3	2	6	3	5	3			4		1		9			
S2	Gin	E3	3	17	3	2	2		1	10		1		1	3		
S2	Gin	E5	1	16	3	5	2			8	1	2		4	5		
S4	Fion	E5	4	17	5	3				15		1	1	4	1		

Note: All episodes in this table are from lessons focused on linear equations

The high incidence of students sharing reasoning is one way in which the observed teachers succeeded in engaging their students in mathematical classroom interactions around demanding tasks. This can be interpreted as an indication of the effect our PD can have on mathematics classrooms. Supporting teachers to enable such student participation was an explicit aim of the project since participation in mathematical classroom interactions around demanding tasks are important learning opportunities for conceptual mathematics and mathematical reasoning.

However, care must be taken when drawing conclusions from this data. Mathematical classroom interaction is often complicated and there are meaningful differences between both the length and the depth of moves. For example, a '1' in Table 4 can represent either a quick one-word reaction from a student or a much longer move from a teacher, which is carefully constructed to support everyone's thinking. Although these are both a single move, they are very different both in character and function.

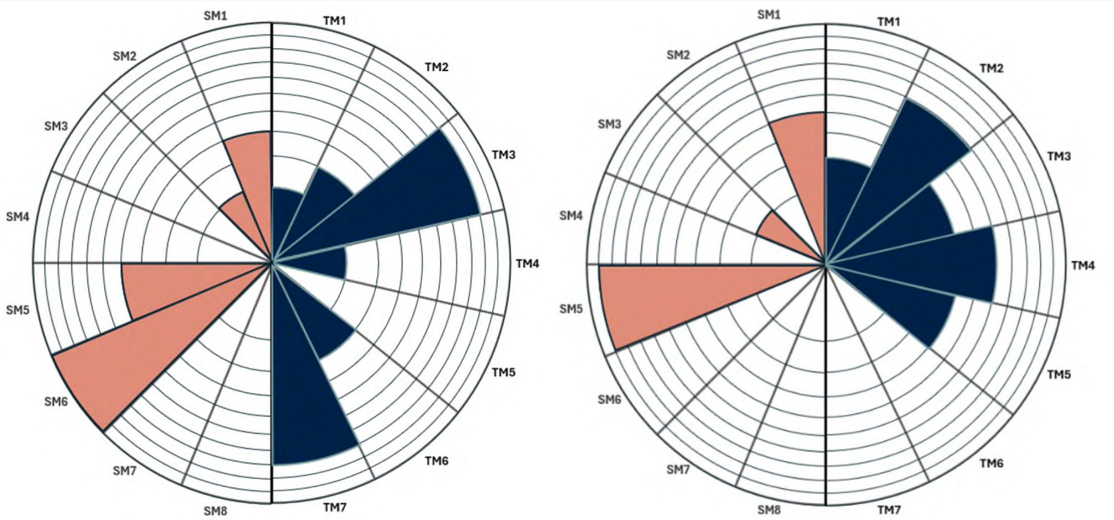


Figure 8a: Alex Episode 1 (x + y = xy) and
Figure 8b: Alex Episode 3 (Always, Sometimes, Never true)

Some patterns in both the teacher and student moves could be connected to the specific task which the class was engaged in. Figure 8a and Figure 8b[2] demonstrate an instance of this difference, illustrating the moves from two different episodes with the same teacher and class. Tasks where students had to engage with the meaning of $x + y = xy$ accounted for almost all cases of TM7 (invite students to read something out loud) and almost all of the instances of SM6. (reading or describing something), with these moves often observed in combination. By way of contrast, the task where students had to determine if statements were always, sometimes, or never true (ASN) typically had more instances of SM5 (explain reasons) with students giving reasons. It is possible that this difference may have been influenced in part by the teachers having ‘practised’ the second of these tasks during a professional development session, but it also speaks to the complementary pedagogical aims of the two activities.

As well as similarities, the coding reported some differences when the same task was used by different teachers. This is unsurprising, as different classrooms involve and invoke different norms and practices. Figure 9a and Figure 9b below represent the moves coded from episodes involving two different teachers tackling the same task.

[2] We have chosen to use an adapted polar area graph for these visualisations, making two notable changes: the radii of the circles have been made proportional to the square roots of the frequencies (so that the area added is constant), and a frequency limit of 10 has been imposed. These changes were intended to limit cases where a dominant move type might obscure the full range of moves.

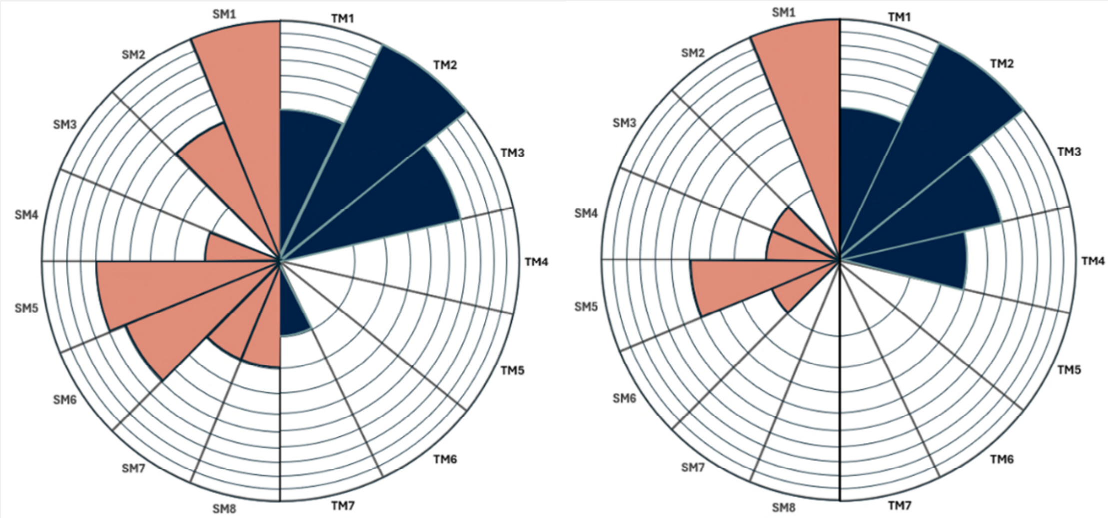


Figure 9a: Ben Episode 1 (x + y = xy)
Figure 9b: Fion Episode 5 (x + y = xy)

At first glance these seem very similar with, for instance, a high number of SM1 (short answer) and TM2 (Understand and connect students' ideas and mathematics; make it accessible to as many students as possible) moves. However, the teacher on the left (Figure 9a) does not explicitly invite participation in demanding discourse (TM4) during this episode – in fact, this was the only video episode from a lesson on linear equations where no TM4 moves were coded. Conversely, the students do ask multiple questions (SM2) in ways that were not observed in the episode on the right (Figure 9b). Whilst coding the researchers also noted some possible differences between mathematical topics, for example how TM4 (teachers encouraging participation in demanding discourse) was less prevalent with angles than with algebra. However, any such differences are likely to have been mediated by the specific tasks and teachers involved, as well as the relative sizes of the data sets, and so are not considered here in detail.

The coding process also included analysing combinations of teacher and student moves. The Sankey diagrams in Figure 10a and Figure 10b below show the relative frequencies of moves which closely followed teacher moves TM2 and TM3.

The move TM2 (understand and connect students' ideas and mathematics; make it accessible to as many students as possible) was followed by a diverse set of moves, possibly due to the breadth of the code: SM1 (short answer) was the most common response, followed by SM5 (explain reasons), SM8 (choral response), SM6 (describe) and then SM3 (explain processes). The short answer code was also the most common follow-up to TM3 (enhance language practices for learning mathematics), but this was closely followed by TM2, suggesting that it is common for teachers to follow up a language-focused move with a mathematics-focused move.

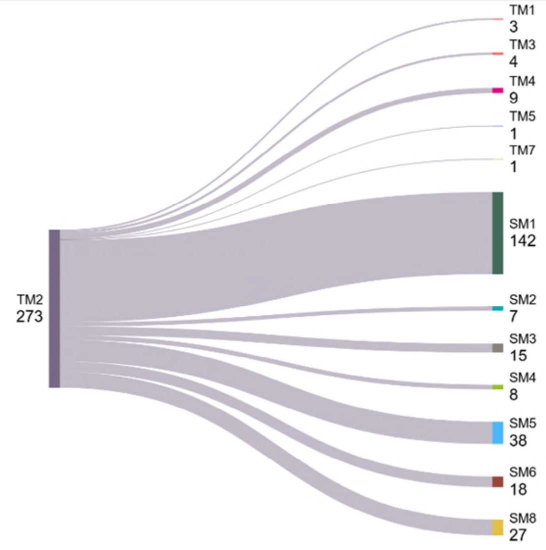


Figure 10a: Sankey Diagram for TM2

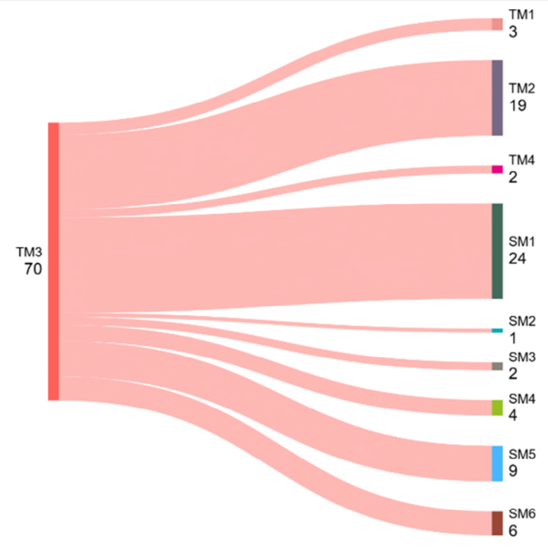


Figure 10b: Sankey Diagram for TM3

Figure 11, the Sankey diagram[3] for TM4 (encourage participation in demanding discourse on mathematics) highlights a different trend.

The shape here is notably different, with the modal follow-up to TM4 being SM5 (explain reasons). This is perhaps not unexpected – the teacher is encouraging participation in demanding discourse, and the students are responding with reasoning and explaining their answers – but it is nonetheless encouraging and speaks to the influence of teacher moves, as well as students' willingness and capabilities if they are explicitly asked to engage in demanding discourse practices.

In some ways, the code of TM4 can be seen as a development of TM2 (and likewise SM5 a development of SM3), such that it is realistic for teachers to embed this move into their practice.

The trends in the Sankey diagrams suggest that this is also profitable; if students are asked not just to give their answer but also to explain, argue or give a reason, they offer a more developed contribution.

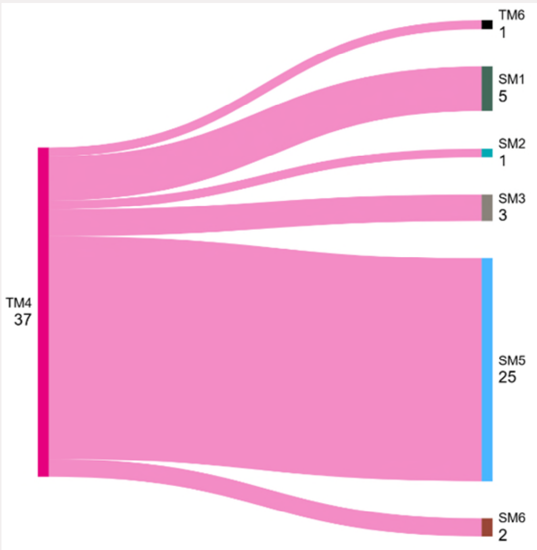


Figure 11: Sankey Diagram for TM4

The data illustrated how this advancement might even happen in the moment, where the teacher uses a TM2 (understand and connect students' ideas and mathematics; make it accessible to as many students as possible), the student responds with an SM1 (short answer), and the teacher counters with a TM4 (encourage participation in demanding discourse on

[3] Diagrams created using SankeyMATIC.

mathematics) to facilitate the student in enhancing their contribution as an SM5 (explain reasons). We observe different variations of this in the moment development. For example, in episode E5 from Fion's algebra teaching, we can observe the chain of moves in Table 5 (we note that we coded student answers as SM5 where they have the structure of explaining reasons even though they might not include a mathematically correct reason).

Table 5: Example of teacher and student move codes

Speaker	Interaction	Code
Teacher:	Does anyone want to offer up their answer, (for?) what they would write? How would you tell someone what was going on in this scenario here? We'll start with you, [name of Student 1…]	TM 4 (encourage participation in demanding discourse on mathematics)
Student 1:	Sometimes.	SM 1 (short answer)
Teacher:	Just sometimes. Can you expand on that?	TM 3 (enhance language practices for learning mathematics)
Student 1:	x plus y equals xy sometimes	SM 6 (describe)
Teacher:	Why? Why sometimes?	TM 4 (encourage participation in demanding discourse on mathematics)
Student 1:	It's cause uhm cause uhm…	SM 5 (explain reasons)
Student 2:	Like, 'cause if it's letters it could be sometimes, but with numbers it's not.	SM 5 (explain reasons)

It is not clear how these trends might develop over time; the data does not capture longitudinal aspects as teachers and students jointly establish classroom norms and practices, and it may be that, in time, expectations might become implicit, such that students are more predisposed to respond to TM2 with SM5. Nonetheless, in terms of our context, the data suggests that attention to making the epistemic and communicative demands of the question clear is likely to have dividends in terms of the ambition and quality of the student response.

We note finally here that there were other examples of longer chains of teacher and student moves in the data which we found encouraging and suggestive of more involved (and evolving) discourse.

Student assessments

The professional development materials were developed to directly address aspects of classroom practice and only indirectly impact student learning. The student assessments were designed to capture this indirect impact on student learning, while the videos were designed to capture the direct effects, i.e. on the quality and quantity of student contributions to the classroom interactions.

As noted previously, not all schools were able to complete tests in all topics and not all teachers were able to complete the test with a comparison class. This resulted in a smaller than planned number of tests overall. In one school and for one topic the assessment suffered from the floor effect where the majority of students answered very few questions. In another school for a different topic the assessment suffered from the ceiling effect where almost all students achieved almost all marks.

The quantitative analysis of the overall test scores showed no significant differences between the intervention and comparison groups in each school and for each topic area. This suggests that there was no direct effect on students' learning in terms of their abilities to answer standard assessment questions. It was also important that there was no detrimental effect which can be an indirect effect where an intervention can reduce the range of aspects of a topic covered during the intervention period.

Each assessment included items where students were asked to give reasons for their answers. In the analysis we looked at any differences in the number of these questions answered as well as the marks awarded for these questions. Again, there were no significant differences between the intervention class and the comparison class in each school and for each topic. This may be a consequence of the PD focusing on oral reasoning rather than written reasoning, or because the development of this reasoning takes more time than available in the lessons involving the project tasks and activities that focused on developing this reasoning.

5. Teacher evaluations

In this section, we report on participating teachers' representations of how they saw their participation in the project and the benefits and utility of the project to their classroom practice, including their perception of changes in their teaching and their students.

The five teachers who attended the final twilight PD session (June 2024) participated in a focus group discussion. All teachers participating in the project had a further opportunity to respond to questions by email (three teachers submitted responses including one who had not attended the PD session). In order to help the teachers feel they could respond openly to questions, the focus group was run by two members of the team who had been least involved with running PD sessions during the project.

Teachers were asked for their views on what difference they felt the project had made to them, what had changed in their classrooms and their practice, which parts of the project they had applied in their own classrooms and which aspects they anticipated still doing in 5 years' time. Questions also asked about the focus of the teachers' discussions about the project with their colleagues.

Three members of the DLRMC team analysed the focus group discussion and further email responses and identified the following themes: developing alternative classroom practices; making connections between topics; focusing more on process than product; and tensions. Verbatim transcripts of the teachers' responses to the questions are used below to illustrate each of these themes.

Developing alternative classroom practices (teachers and students)

Indicators of how teachers see the project having impacted their practice or mediated some changes in them or their students in the classroom included the use of the mathematics register and its associated features of formality and precision, with teachers reporting having become more conscious of their own language use and connecting this with shifts in students' expectations of mathematics classroom language. The teachers also reported that their lessons involved richer and deeper classroom discussions.

Teachers related their own use of the mathematics register to their being more conscious about the phrasings they used themselves, for example, reassessing multiple phrasings and phrasings students might use. Teachers also connected this awareness with the mathematics, for example, '*Especially in topic areas like algebra, I was kind of just using the language that I would normally use rather than actually thinking, Oh, that actually needs more explanation or that needs some exploration*' [Fion], and '*It has helped me focus on the language and nuances that pupils deploy to explain their learning and how conceptual errors can be perpetuated if not addressed.*' [Ikuya].

Another teacher also reported using revoicing or encouraging students to repeat their contributions with tweaks to students' language until students seemed happy with the formal and precise language. They compared recent lessons with those from the start of the year, noting that '*we don't often get stuck for the word that we need to use*' [Alex], which they attributed to students having become used to using the appropriate mathematical words, including in student-student interactions where they might previously have used more colloquial language. Fion noted that students had got used to the ways of working in the project, which included using the more formal language alongside their more colloquial language and moving between these.

Teachers reported changes to richer and deeper classroom discussions, which some related to changing expectations of students, including those identified in schools as '*lower ability*'. Teachers commented on a change in their expectations of students in classroom discussion, noting that students were taking arguments further:

'So for angles, sometimes they normally can find the answer, but it's now exploring that depth. Why have you chosen that answer? And then being able to use, as [teacher name] said, the correct vocabulary and also have different arguments with me about some of the answers they have and so the discussion is much deeper and richer' [Jackie].

Other teachers commented on the affordances of intervening less, to give students opportunities to give explanations and share their reasoning, and affordances of listening more to students. One connected a shift in their own awareness of language use to listening as part of their classroom practice, explaining that:

'In one of the sessions [PD workshops] we did something we were talking about listening for and listening to so I'm more conscious and deliberately making those effort in a lesson to listen for and listen to the students.' [Gin].

Another teacher commented that listening to students gave indications of how they could stretch students, noting surprise at how far they could push students with the right support. Working on the demanding algebraic tasks had indicated to this teacher that they may have been limiting students by having low expectations.

Making connections between topics

Teachers reported that they were both making more connections between topics now and also that this had a higher priority in their teaching. Teachers commented that connections between topics had been aided by work on the project tasks, identifying that they would not previously have connected geometric work on parallel lines with transformations, and another connected probability with the ASN algebra task

'I just did algebra and probability and it's linked up. And I realised that the students were more into it because we've just gone through the scheme of work on probability and brought in the Always, Sometimes and Never and that gave them a deeper understanding of what they did on algebra.' [Hiro].

Focusing more on process than product

One theme that occurred repeatedly in relation to other points noted above was a shift from focusing on completing tasks, which they called a product perspective, to the process of working on the tasks. Thus, instead of a product of a task as an end goal and thinking they must get to a certain point, they now focused more on *'what's the process of getting there'* [Alex] and *'what are the processes such as connections, thinking and reasoning that are sometimes overlooked?'* [Gin].

As one teacher noted:

'If your eyes are on the end goal, then you often cut down the justify explain because you're like Oh I've only got 10 minutes left and I must get that by the end of the lesson, so you sort of start dragging them down the path with you'

which they felt limited opportunities for conversation and exploration of tangents and impacted opportunities for developing explanations and language use:

'So you often don't get the explanations and the language because you sort of sometimes accept that bad language because it just gets us there.' [Alex].

Tensions

Teachers' discussion of the points above also raised issues of tensions in their practice. Several commented on their familiarity with some of the practices discussed in the PD discussions, for example:

'It's just a reminder of the good practices that we should have been using in the first place...I think that most of the things that we did in the programme are things that I knew I should be doing' [Gin].

However, despite this familiarity, the conversation indicated the teachers' awareness that these practices were not established in their daily teaching. Some connected their own move away from these practices with constraints and priorities of working in their school system. For example:

'I think for all of us, we did this in our training, we started out teaching like this and as we've gone through the years, we've become like Oh, we need to deliver this product, we need to have certain things in our books, we need to have this. The expectation has driven us away from some of these practises, so it's been nice to revisit it now with our knowledge of that expectations that's there and seeing that we can fit all of this in and still meet those expectation and actually exceed them in some cases because the kids are now able to answer proof questions which sometimes before they wouldn't have been able to do.' [Jackie].

All six teachers who responded to the evaluation either in the focus group or via email talked positively about changes in their practice that they attributed to their participation in the project. Mindful of research that highlights the challenges of changing practice and sustaining this change, we interpret these responses cautiously, as an opening for change rather than finished work. Specifically, we see it as an opening for more work by and with these teachers to support these changes becoming well-established in their practice. Indicators of teachers' intentions to sustain the changes they identify include embedding DLRMC tasks in their schemes of work and making connections beyond these tasks. One teacher commented:

'I think for me going forward beyond those five years that you mentioned is for me now just go back and look and say, OK, these are best practices, these are better ways to actually develop this particular concept and it's not just the three or four that we looked at now, but I can now take this to other topics later

on. So I can see this being extended to multiple topics and multiple concepts because the key is connections and the key is that the language, the key is the reasoning and so on. So for me, I think going forward beyond those five years is now ensuring that we put those best practices into place.' [Gin]

Extending beyond the practice of these teachers, one teacher reported discussing approaches used in the DLRMC project with a less experienced colleague. They had discussed different phrasings and how teachers could use their own linguistic resources to support students' understanding alongside discussing approaches that create opportunities for students to work on less structured tasks.

6. Conclusion

The project aimed to develop a professional development programme for secondary mathematics teachers to address linguistic challenges in learning mathematics. It also researched the impact of this programme on teachers' classroom practices and students' learning, focusing on three Key Stage 3 topics: linear equations, angle properties of parallel lines, and introductory probability.

Language-responsive mathematics teaching integrates and enhances both mathematics and language, using literacy strategies focusing on mathematical communication, and incorporating informal and home languages, and teacher moves that support classroom discussion.

The professional development programme incorporated mathematically demanding tasks that encourage reasoning and explanation. The programme focused on four key aspects of language-responsive mathematics teaching (Figure 1):

1. Identifying the linguistic demands of the specific topic
   This includes topic-specific meaning-related language and argumentation that emphasises mathematical concepts and relationships.
2. Listening and feedback
   This included distinguishing between listening for and listening to, as well as developing norms and routines that support mathematical communication, using tasks and teacher moves.
3. Developing student reasoning and explanations
   This was achieved through the use of mathematically demanding tasks and attention to the linguistic forms of appropriate arguments.
4. Making connections
   These connections could be between multiple representations that illustrate mathematical structures and concepts, or between different mathematical topics, or between different reasons and explanations.

Limitations

We recognise here multiple limitations and possible sources of bias subsequent from both the makeup and the handling of the data. As is common in this type of research, the participating teachers were not always able to attend each session or were not able to complete all aspects of the project. These were often due to clashes with school commitments such as staff meetings or parent meetings. Some teachers also left the project early or started later than others due to challenges securing consent from all those involved, shifts in school priorities and policies, or teachers changing schools. Consequently it was not possible to collect all of the data included in the research plan.

The project planned to limit the disruption to learning, so opportunities to generate classroom data were limited by teachers' choice of tasks and how these fitted with their schemes of work. As a result, some tasks have been studied in more detail than others, but this provided opportunities for comparison of task use between teachers.

Additionally, the episodes in Stage 3 of the coding had already been selected as being of interest in the previous stage of coding, which meant that for Stage 3 coding there were more episodes involving some teachers than others. Further, as noted above, mathematical classroom interaction is often messy, and categorical coding can neglect nuance. Nonetheless, these results provide a broad view of teacher and student moves which supports some preliminary conclusions. Further research is needed to examine these conclusions over time and across other topic areas and age groups.

Recommendations for further research

This project was a relatively small development project resulting in the production of easy-to-use materials to support the development of language-responsive mathematics teaching. These materials have their basis in existing research, but much of this research has been conducted in other countries or with younger children.

Further research is needed to examine the impact of these design principles and teacher moves on the structure and nature of students' reasoning. While this project and other research shows that the amount of student reasoning in lessons increases with language-responsive mathematics teaching, the nature of this reasoning has not been examined. As part of this project there was a focus on considering structures of explanations and arguments and not just vocabulary, in terms of teachers' awareness of these structures, but also in terms of the quality of students' explanations and arguments. While there were several examples of high-quality student explanations and arguments in the video data and assessment data, further research is needed into what supports students in developing these.

This project also focused on three key topics within the Key Stage 3 mathematics curriculum. These topics were chosen as they involved underlying concepts that are fundamental to understanding mathematics and contrasting styles of argument structure and use of representations. While some aspects of language-responsive mathematics teaching carry across different topics, meaning-related language, some argument structures and meaning-related representations are usually specific to the mathematical concept or process. Further research is needed to identify these aspects of classroom discourse for other topics in the secondary curriculum.

Suggestions for further implementation

The professional development materials developed address the need for mathematics-focused approaches to professional development as wider research has shown that effective practices are often at the level of a curriculum topic. With the demands on teachers, schools and students, flexible, tailored professional development such as the materials developed can help to address the difficulties encountered by many teachers and schools in accessing high-quality mathematics-specific professional development.

Many of the features of language-responsive mathematics teaching are easily embedded within existing resources that support mathematics teaching and learning. As departments continue to develop the resources they use and the classroom practices they are focusing on, the guidance offered as part of the professional development materials can support the wider implementation of language-responsive design principles.

The materials developed as part of this project are freely available at:

https://www.education.ox.ac.uk/project/developing-language-responsive-mathematics-classrooms/

References

Abedi, J., & Herman, J. (2010). Assessing English language learners' opportunity to learn mathematics: issues and limitations. *Teachers College Record*, *112*(3), 723–746.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J. N. N., Planas, N., Setati-Phakeng, M., Valero, P., & Ubillús, M. V. (Eds.). (2016). *Mathematics education and language diversity: The 21st ICMI study*. Springer. https://library.oapen.org/handle/20.500.12657/48204.

Bell, C. A., Qi, Y., Witherspoon, M. W., Howell, H., & Torres, M. B. (2020). *The TALIS Video Study observation system*. Organisation for Economic Co-operation and Development. https://www.oecd.org/en/about/projects/global-teaching-insights.html#technical

BERA. (2018). *Ethical guidelines for educational research* (4th ed.). https://www.bera.ac.uk/researchers-resources/publications/ethical-guidelines-for-educational-research-2018

Clark-Wilson, A., Hoyles, C., Noss, R., Vahey, P., & Roschelle, J. (2015). Scaling a technology-based innovation: windows on the evolution of mathematics teachers' practices. *ZDM – Mathematics Education*, *47*(1), 79–92. https://doi.org/10.1007/s11858-014-0635-6

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher*, *32*(1), 9–13.

Drageset, O. G. (2015). Different types of student comments in the mathematics classroom. *Journal of Mathematical Behavior*, *38*, 29–40. https://doi.org/http://dx.doi.org/10.1016/j.jmathb.2015.01.003

Drageset, O. G. (2021). Exploring student explanations: What types can be observed, and how do teachers initiate them? *Nordic Studies in Mathematics Education*, *26*(1), 537–572. https://ncm.gu.se/wp-content/uploads/2021/04/26_1_053072_drageset.pdf

Erath, K. (2017). *Mathematisch diskursive Praktiken des Erklärens. Rekonstruktion von Unterrichtsgesprächen in unterschiedlichen Mikrokulturen [Mathematical discursive practices of explaining. Reconstruction of classroom conversations in different microcultures]*. Springer Spektrum. https://doi.org/https://doi.org/10.1007/978-3-658-16159-0

Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: a review of the state of development and research. *ZDM – Mathematics Education*, *53*(2), 245–262. https://doi.org/10.1007/s11858-020-01213-2

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms: the case of explaining to learn and learning to explain. *Educational Studies in Mathematics*, *99*, 161–179. https://doi.org/10.1007/s10649-018-9830-7

Heller, V., & Morek, M. (2015). Academic discourse as situated practice: An introduction. *Linguistics and Education*, *31*, 174–186. https://doi.org/10.1016/j.linged.2014.01.008

Ingram, J., Lindorff, A., McCann, E., Riggall, A., & Sani, N. (2020). *TALIS Video Study National Report*.

Ingram, J., Stiff, J., Cadwallader, S., Lee, G., & Kayton, H. (2023). *PISA 2022: National Report for England*. DfE.

Kaye, N. (2023). The cumulative impact of socioeconomic disadvantage on educational attainment during austerity: a comparative cross-cohort approach. *Oxford Review of Education*, *50*(2), 186–206. https://doi.org/10.1080/03054985.2023.2234287

Lenz, K., Obersteiner, A., & Wittmann, G. (2024). Who benefits most from language-responsive learning materials in mathematics? Investigating differential effects in heterogeneous classrooms. *Educational Studies in Mathematics*, *116*(2), 185–211. https://doi.org/10.1007/s10649-024-10321-9

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge.

Mason, J. (2006). Being mathematical with and in front of learners Attention, awareness, and attitude as sources of differences between teacher educators, teachers and learners. In B. Jaworski & T. Wood (Eds.), *Handbook of mathematics teacher education* (pp. 31–56). Sense.

Mason, J. (2011). Classifying and Characterising: Provoking Awareness of the Use of a Natural Power in Mathematics and in Mathematical Pedagogy. In O. Zaslavsky & P. Sullivan (Eds.), *Constructing Knowledge for Teaching Secondary Mathematics: Tasks to enhance prospective and practicing teacher learning* (pp. 39–55). Springer.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically* (Second Edition ed.). Pearson.

Moschkovich, J. N. (2015). Academic literacy in mathematics for English Learners. *The Journal of Mathematical Behavior*, *40*, 43–62.

O'Halloran, K. (2005). Intersemiosis: Meaning across language, visual images and symbolism. In *Mathematical discourse: Language, symbolism and visual images* (pp. 159–188). Continuum.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge and Kegan Paul.

Pöhler, B., & Prediger, S. (2015). Intertwining Lexical and Conceptual Learning Trajectories - A Design Research Study on Dual Macro-Scaffolding towards Percentages. *EURASIA Journal of Mathematics, Science and Technology Education*, *11*(6), 1697–1722. https://doi.org/10.12973/eurasia.2015.1497a

Prediger, S. (2022). Leveraging and connecting conceptions of amount and change: A content-specific approach to adaptive teaching practices. *The Journal of Mathematical Behavior*, *66*. https://doi.org/10.1016/j.jmathb.2022.100970

Prediger, S., Erath, K., Weinert, H., & Quabeck, K. (2022). Only for Multilingual Students at Risk? Cluster-Randomized Trial on Language-Responsive Mathematics Instruction. *Journal for Research in Mathematics Education*, *53*(4), 255–276. https://doi.org/10.5951/jresematheduc-2020-0193

Prediger, S., & Neugebauer, P. (2021a). Can students with different language backgrounds profit equally from a language-responsive instructional approach for percentages? Differential effectiveness in a field trial. *Mathematical Thinking and Learning*, *25*(1), 2–22. https://doi.org/10.1080/10986065.2021.1919817

Prediger, S., & Neugebauer, P. (2021b). Capturing teaching practices in language-responsive mathematics classrooms: Extending the TRU framework “teaching for robust understanding” to L-TRU. *ZDM – Mathematics Education*, *53*(2), 289–304. https://doi.org/10.1007/s11858-020-01187-1

Schleppegrell, M. J. (2007). The Linguistic Challenges of Mathematics Teaching and Learning: A Research Review. *Reading & Writing Quarterly*, *23*(2), 139-159. https://doi.org/10.1080/10573560601158461

Watson, A., Jones, K., & Pratt, D. (2013). *Key ideas in teaching mathematics: Research-based guidance for ages 9-19*. Oxford University Press.

Appendix A: Details of Stage 3 coding

The tables below offer further detail to clarify and illustrate the codes used in the Stage 3 coding process. A fuller working document was generated by the researchers, but this is not reproduced entirely here for reasons of space.

The researchers followed some general principles:

- Only sections of episodes involving whole class discussion should be coded (with the slight exception of a covering TM1 code if the teacher is circulating – see below).
- Moves which are to do with classroom management over and above the mathematics should not be coded.
- A single teacher move may include multiple features within the same code. For example, a teacher might write up a student's verbal contribution as algebra (making it public and connecting representations) and immediately use this as the basis for a guiding question. This would be coded as one TM2.
- Codes should absorb features of speech such as echoes, set-up and simple repetitions; in such instances it is not intended that each utterance is coded separately. For example, if the teacher repeats a question when the learner mishears or is buying time, and there is no substantial reformulation, this should be allocated a single code.

Speech in classrooms can be messy. In a small number of cases where the intent of a speaker was clear, interrupted or incomplete moves were coded in the same way as full moves, even though these moves may have been considered differently if the focus was more normative.

The descriptions and examples of TM1 to TM6 in the table below are taken from Erath et al. (2021) and those of SM1 to SM6 are drawn from Drageset (2015, 2021), Erath et al. (2018) and Erath (2017).

Table 6: Details of teacher moves TM1 to TM7

Code	Description	Examples	Clarification Notes, Instances and Non-instances
TM1	Plan and prepare collective discussions that focus on mathematical concepts	• Use tasks suitable for enabling discussions • Anticipate student responses during planning • Monitor students' processes during individual/group work • Purposefully select and sequence ideas that are presented • Tailor discussions to their epistemic function	• The planning and preparation of tasks can often be inferred but not directly seen in episodes. Code the use of a task once at the beginning of the episode or whenever a new task is introduced. Monitoring students' responses can be coded as TM1, but only when the teacher is both moving around the classroom and actively interacting with learners in a way that moves beyond redirection or instruction, and it can be sensibly inferred that these interactions might inform future collective discussion. For example, in one instance, after some minutes of individual work on finding connections in a given figure with angles in parallel lines the teacher asks students to use their mini whiteboards: "I want you to write one or more statements or equations. If you give me a statement or equation that no one else has, I will give you a merit. Okay, so have a think […] So think about what is your most unique answer, 'cause I want to try and get as many of these as possible." However, the following instance does not qualify, as it is a reminder focused on classroom management: Teacher (walking around the room): "make sure you write those examples down".
TM2	Understand and connect students' ideas and mathematics; make it accessible to as many students as possible	• Explore the details of children's thinking and pose informed probing questions • Use guiding questions providing directions and promoting confidence • Be responsive and adjust challenges to students' mathematical and linguistic capabilities in the moment • Use gestures, drawings, connect	The principal focus of this move is the mathematics. If revoicing is used, it typically reinforces the mathematics. "So, what are you making t equal to?" "Can anyone give me another example where this doesn't work?" "What does outcome mean to you?" "What makes two things congruent to each other?" "Fractions and *multiples*? We're thinking types of numbers…"

Code	Description	Examples	Clarification Notes, Instances and Non-instances
		representations and language varieties as supports • Keeps student in the role of the responsible speaker • Revoice • Explicate epistemic demands • Choose the language that represents a mathematical idea more transparently	
TM3	Enhance language practices for learning mathematics	• Explicate communicative demands • Revoice • Reformulate, paraphrase • Initiate self-repair, ask for clarification/elaboration • Pointing to supporting materials, make use of connecting language varieties and representations • Provide vocabulary and syntactical structures, complement words • Act as a language model • Elicit students' languages	The principal focus of this move is language. If revoicing is used here it markedly brings attention to the language. TM3 is deployed to develop or highlight some element of the language register/repertoire. "So, it wouldn't be equal, it would be… it's on about *true* or *false*." "If it's somewhere in the middle, how could I say it's not always true, but it's…" "Anyone got any further, any other way of saying that?" "Why does this [underlines something on the board] make it an explanation? Why does like adding in the word because make it an explanation?" "If things don't equal, you can do this *(draws on board)* to show does not equal, that's the sign for does not equal. So you might want to use that – it's an equals with a line through it." "And that is how you need to be writing your reasoning – we're going to use the word 'because' because you're explaining a reasoning, we're not making a statement, co-interior angles sum to 180, you know that, you're writing because… you are making sure you explain your reasoning."
TM4	Encourage participation in demanding discourse on mathematics	• Use inquiry questions • Invite students to explain, argue, give reasons,	The word 'demanding' in the description is key here – this does not include simple narration or description. So "Have a chat, what do you think?" does not qualify. However, the following

Code	Description	Examples	Clarification Notes, Instances and Non-instances
		etc. • Focus on students' mathematical ideas rather than encouraging every contribution	are instances of TM4: "I don't want you to move on to the second, to the next card until you've looked through the first one and decided whether it's *always*, *sometimes* or *never*. Convince each other that is the case, for each one." "You've now discovered the answer to this. How do you think you'd tell someone else what this means, what the answer to this question would be?"
TM5	Pay attention to your feedback and evaluation of students' mathematics	• Emphasize joint examination of the quality of students' ideas; establish shared criteria • Handle mistakes and nescience positively and in a future-oriented way • Repeat correct student utterances • Initiate self-repair	This does not include simple repeats of student utterances. TM5 is intended to code moves where correct answers or mistakes are explicitly highlighted by the teacher in a way that moves the discussion on – otherwise it may better be coded as TM2 or even TM3. (After a student volunteers 1 and 1 as a possible answer, the teacher addresses the class as a whole as she writes on the board.) "Let's try it. One plus one, does it equal one times one?" "Yeah… is that enough to tell me they're parallel? If I drew two lines like this (draws two lines on the board) is that enough to tell me that they're parallel?" "Does anyone have something to add to that? [Name] says b is equal to d, and horizontally opposite. Do we agree? What do you guys think?"
TM6	Purposefully use pauses and silences	• Pause to allow students to articulate their thinking • Listen to students' contributions	Listening to students' contributions or having a brief pause does not automatically earn this code; rather it suggests significant engagement with what the student has said, or a meaningful amount of time being given to individuals to support articulation. For example: the teacher pauses the class discussion to allow students time to think, and possibly write down their ideas: "If you need to write something on your paper to determine the conclusion, go ahead… we don't want to guess, we're not guessing, there has to be a reason for your response."
TM7	Invite learners to read something out loud	• Read out statement on the board	This move was gained from the data. It grasps teacher moves that do not ask for explaining or reasoning but for verbalising something that is visual in the

Code	Description	Examples	Clarification Notes, Instances and Non-instances
		• Refer to signs in a diagram	classroom. (Talking about an algebraic statement on the board.) “So, what does it mean in words?” “Who wants to tell me what’s written on the board?”

Table 7: Details of student moves SM1to SM8

Code	Description	Examples	Clarification Notes, Instances and Non-instances
SM1	Short answer	• Yes/No • Result (e.g. number) • Single technical term (e.g. “triangle”)	Most short answers can be coded as SM1. Instances include: “Two and two.” “Sometimes.” “When y equals one.” However, when an answer was grounded in a description of something visible, this should be considered against SM6.
SM2	Question	• Further enquiry (e.g. for understanding a question or assignment) • Question starting a new sequence/initiation	Instances include: “Sir – can x and y be the same?” (In an angles lesson, after a sequence where the teacher was looking for pairs of angles of equal size, and had moved on, suggesting that the class had exhausted the possibilities.) “Wouldn’t e and c be one?”
SM3	Explain processes	• Explaining – HOW • Sequenced • Explaining a general procedure • Explaining a concrete calculation	In these instances, the learner offers how they arrived at a result, perhaps by sharing a calculation: “If t equals two, you do two times by two which is four, take away three that’s one…” However, in instances where the student establishes a comparison or builds from a calculation, this should be coded as SM5 – such as was the case in the following instance: “Because if p equals four, that would be greater than nine add p, but if p equals one, it wouldn’t be greater than nine add p.”
SM4	Explain concepts	• Explaining – WHAT • Not sequenced • Can refer to other concepts	Here the student attempts an explanation of a mathematical concept. “If it’s fair – it’s fair if(?) always a chance you can win or lose.” “That symbol means it – it’s saying that 4 p is larger than 9 plus p.”

		• Can refer to examples and non-examples • Can refer to diagrams or conceptions/mental models	
SM5	Explain reasons	• Explaining – WHY • Not sequenced, “needs” logical connectors • Can refer to concepts or propositions • Can refer to examples and non-examples • Can refer to general procedures • Can refer to diagrams or conceptions/mental models	This should move beyond explaining a concrete calculation and have some element of reasoning; if a calculation (how) is given as a reason with no additional language content (why) at the front and the end, this should be coded as SM3. “Because you can’t really change the value of q... because if q equals one, then you can’t change it halfway to suit the question.” “Because it has an add, not a multiply.” “Because it [a student reasoning] does not tell anything about them being equal." “Because co-interior angles add up to 180 degrees."
SM6	Describe	• Reading out an equation • Describing a diagram • Refers to often visually obvious things • Can be sequenced	This can sometimes appear as an extended version of move SM1. However, in each case the description or simple statement should be based on an observation of something ‘visible’. “x plus y equals xy” (student reads out loud the algebraic statement written on the board) (The teacher has asked how the students know that two lines on a diagram are parallel.) “’Cause arrows going through them.”
SM7	Engage in a teacher-style move	• Evaluating others • Prompting others	This move was gained from the data. It refers to student utterances in which they engage in a way that is typically associated with teacher moves. Sample instances include: (One student speaking to another but audible to the whole classroom) “Just read what you see!” A student corrected the teacher on their use of ‘minus seven’ instead of ‘negative seven’, shouting out “You said it, sir!” (This prompted a TM3.)

SM8	Choral Response	• Using mini whiteboards • Using hand signs • Collectively answering a SM1 to a teacher move	This move was gained from the data. The class responds in a definite way to a teacher question with a short answer, acting together but as individuals. For example, they might all volunteer answers on mini whiteboards at the same time, or put their hands up to vote for an answer to a multiple choice question. This does not however include responses to rhetorical questions from the teacher, or cases such as those where a small number of students respond in an unclear way (for example murmuring assent).

Appendix B: Stage 3 coding full results

This table gives the full results for Stage 3 coding of the 20 episodes on linear equations (LE), probability (PR) and angles (AN).

Table 8: Frequency of codes

Sch.	Teacher	Episode	TM1	TM2	TM3	TM4	TM5	TM6	TM7	SM1	SM2	SM3	SM4	SM5	SM6	SM7	SM8
1	Ben	LE E1	4	13	6				1	15	4		1	6	5	2	2
1	Ben	LE E2	1	10	8	1			1	13		2			1	1	
1	Ben	LE E3	1	17	7	9	1			16		1		11			17
1	Alex	LE E1	1	2	8	1		2	7	3	1			4	10		
1	Alex	LE E2	2	22	7	4				17	1	4		13			
1	Alex	LE E3	2	6	3	5	3			4		1		9			
1	Alex	LE E4	2	16	6	3	3			6		5	1	13			
2	Hiro	LE E3	1	14	2	2	2		2	12	2	4		2	3		3
2	Gin	LE E3	3	17	3	2	2		1	10		1		1	3		
2	Gin	LE E5	1	16	3	5	2			8	1	2		4	5		
4	Fion	LE E5	4	17	5	3				15		1	1	4	1		
2	Hiro	PR E4	1	9	1					7			2	1			
2	Hiro	PR E6	1	12	2	1				9		1	2	3			
2	Hiro	PR E7	2	18	1					21		1	1				

4	Fion	AN E3	1	4			3			2					4		
4	Fion	AN E4	2	4				1		2					1		
4	Fion	AN E5	6	30	2	0				18	1		2	6			1
4	Fion	AN E7	6	9	7	3				4				5			1
4	Fion	AN E8	1	11	2		5			10			1		1		3
5	Ikuya	AN E2	2	32	11		7			28	2		1	4	7		
Total			44	279	84	39	28	3	12	220	12	23	12	86	41	3	27

UNIVERSITY OF
OXFORD

Oxford University Department of Education
15 Norham Gardens, Oxford OX2 6PA
www.education.ox.ac.uk